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A Theoretical Mathematical Model for Exploring Learning Dynamics: A Scenario-Based Analysis

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Abstract This work offers a theoretical reflection on interactions between students and teachers through the development of four modelled scenarios. Using mathematical tools, it seeks to analyse, in an abstract manner, the dynamics of learning and how they evolve in response to different forms of pedagogical intervention. Each scenario is designed as a conceptual illustration highlighting essential dimensions of teaching and learning, such as feedback, adaptability and personalisation. The aim is not so much to study actual practices as to show, through modelling, how these mechanisms can be understood and interpreted. This work thus aims to enrich the theoretical understanding of educational processes and highlights the contribution of formal models as a framework for analysing the complexity of pedagogical interactions.

Keywords Education, mathematical model, modelling, pedagogical intervention, dynamics of learning

AMS 2010 subject classifications 93A30, 34A34 , 97M10

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1. Introduction

Education is a complex and dynamic field in which researchers, practitioners and policy-makers are increasingly interested. Understanding learning and teaching processes is essential for improving pedagogical practices and promoting quality education for all learners. With this in mind, modelling the interactions between teachers, students and learning content provides a valuable framework for analysing and interpreting these complex dynamics. Our study fits within this perspective. Drawing on the foundational works of renowned researchers such as [5], [30], [1] [20] [12] [26] [19] [18] [24] and [16], as well as the major contributions of educational theorists such as [28], [21] [23] [29] [4] [22], and [2], [27] and the Mathematical Modeling in Mathematics Education as [17], we propose an in-depth exploration of four teaching and learning scenarios. [5]'s research on the culture of education highlights the importance of the educational environment in the learning process, while Vygotsky's theories on socio-cognitive development emphasize social interaction as a key driver of learning. Furthermore, Dewey's ideas on experiential learning and progressive education provide a conceptual framework for understanding the dynamic nature of learning. [28]'s work on cognitive development and [22]'s theory of multiple intelligences offer complementary perspectives on the diversity of students' abilities and learning styles. Additionally, [2]'s social learning theory underscores the critical role of modeling in the acquisition of skills.
[17]'s work on Mathematical Modelling in Mathematics Education, they introduces a continuous-time

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mathematical model to analyze student's interactions with mathematics, [27]' research on Mathematical Reasoning (MR) in high school mathematics education in Morocco by analyzing curricula and official textbooks for the first two years by using a commognitive framework and a quantitative approach. [25] are working on analysing the interdisciplinary benefits of modelling in mathematics education. These modelling problems can be used by teachers at all levels to improve the service offered to pupils in their classrooms, through associated problem-solving analysis activities that help students achieve the highest learning standards. With a continued focus on developing critical thinking, creativity, subject excellence, and preparation for higher education or professional life, mathematical modelling is positioned as an innovative, research-validated method for cultivating these essential skills. This is an essential direction that mathematics education must take to prepare students for the future. [7] discussed five perspectives on research and practice in teaching and learning mathematical modelling in mathematics classes from nursery school to sixth form, and encouraged participants to deepen our understanding of teaching and learning mathematical modelling.
In this study, we will detail the four teaching and learning scenarios analyzed, highlighting the objectives of each case study and their underlying theoretical concepts. We will also examine the relevance of modelling in the field of education and explain how it can enhance our understanding of learning dynamics. Finally, we will present an overview of the methodology used in our study, highlighting the modelling tools and techniques used to analyse the data and draw relevant conclusions about teaching practices.

2. Introduction of the mathematical model

The propagation and correction of learning errors in the classroom are fundamental aspects of any educational process. Here is a general model for understanding this process

- Identifying errors: Teachers must be careful to identify the errors that students make in their learning. This can be done through formative assessments, classroom observations or individual discussions with students.
- Error analysis: Once errors have been identified, it is important to analyse them to understand their underlying causes. Is it due to a lack of conceptual understanding, an error in reasoning, gaps in prior knowledge, or other factors?
- Immediate feedback: Ideally, errors should be corrected immediately after they occur. This allows students to understand their errors as soon as they occur, making it easier to correct and consolidate concepts.
- Individualisation of learning: Understanding that each student may make different mistakes for different reasons is crucial. Teachers need to adapt their approach to meet students' individual needs, providing extra support where needed.
- Correction strategies: Teachers can use a variety of strategies to correct errors, such as additional explanations, concrete examples, additional practice exercises, remedial activities, or even the use of alternative teaching resources.
- Monitoring and evaluation: It is important to monitor students' progress after errors have been corrected to ensure that they have correctly assimilated the concepts. This can be done through formative or summative assessments, classroom observations or individual discussions.
- Repetition and consolidation: Correcting errors is often not a one-off process, but rather an iterative one in which students need several opportunities to practise and consolidate their knowledge in order to completely overcome their mistakes.

By incorporating these stages into their teaching practices, teachers can help pupils to learn constructively from their mistakes and progress along their learning path.

3. Mathematical model

To develop a mathematical model for propagating and correcting learning errors in the classroom, we can use concepts from learning theory and information theory. Here is a simplified example of a mathematical model:

Suppose we have a set of students $E = \{e_1, e_2, \dots, e_n\}$ in a class, and each student e_1 has a set of skills or knowledge $S_i = \{S_1, S_2, \dots, S_n\}$.
We can represent a student's knowledge in the form of a binary vector, where each element of the vector indicates whether or not the student has mastered a specific skill.For example, $e_i = (1, 0, 1, 1, \dots, 1)$ means that student e_i has mastered skills $S_{i1}, S_{i3}, \dots, S_{im}$ but not S_{i2}. Now, suppose that students are exposed to learning activities, such as lectures, exercises or homework. Each activity A_j can be associated with a set of required skills R_j, also represented by a binary vector indicating the skills needed to pass that activity. When a student takes part in an activity, they may make mistakes due to gaps in their knowledge. We can model the probability of a student ei making a mistake during the A_j activity using a function that depends on the differences between the student's skill vector and the skill vector required for the activity. For example, a similarity function such as Hamming distance can be used. Once errors have been identified, teachers can provide feedback in the form of corrections. We can model the effectiveness of the correction by adjusting the students' skill vectors according to the errors made and the feedback provided. For example, a simple approach would be to increment the elements of the skills vector for the skills required in the activity where a mistake was made. This model can be iteratively applied to each learning activity in the classroom, and students' skills can be updated over time based on their interactions with the activities and feedback from teachers. Of course, this model is a simplification and can be extended to take account of various factors such as long-term retention of knowledge, individual differences in learning rates, etc. But it does provide a mathematical basis for assessing the impact of learning on learning outcomes. But it provides a mathematical basis for understanding the propagation and correction of learning errors in the classroom. To include all the factors that could influence students' learning processes in our mathematical model, we need to consider several aspects, such as long-term retention of knowledge, individual differences in learning styles and learning rates, social interactions, motivational and emotional variables, and the teaching strategies used. Here is an extension of our mathematical model that incorporates these factors:

- Long-term retention of knowledge: We can introduce parameters to model the durability of knowledge acquired by students over time.For example, each time a student masters a skill, that skill could be reinforced over time with a decreasing probability of being forgotten.
- Individual differences in learning styles and learning rates: Each pupil could be characterised by individual parameters which influence their learning, such as their ability to assimilate new information, their preference for certain types of learning activities ...
- Social interactions: Interactions between students and with teachers can also be modelled. For example, a student might benefit from collaboration with peers or be influenced by the behaviour of other students in the class.
- Motivational and emotional variables: Variables linked to students' motivation and emotions could be included, such as interest in the subject, self-confidence, stress levels, etc. self-confidence, stress levels, etc. These variables could affect the likelihood of making mistakes and receptiveness to teacher feedback.
- Teaching strategies : The teaching strategies used by teachers can also be taken into account. For example, some teachers might provide more frequent or more detailed feedback than others, which could influence the students' learning process.

By combining these factors in a mathematical model, we could obtain a more complete and realistic representation of the learning process in the classroom.
Let $E = \{e_1, e_2, \dots, e_n\}$ be the set of students in the class, and for each student ei, we have a set of skills $S_i = \{S_1, S_2, \dots, S_n\}$. We can represent a student's knowledge in the form of a binary vector as before, $e_i = (1, 0, 1, 1, \dots, 1)$.
To take into account the different factors influencing learning, we could extend this model by introducing additional parameters for each student, such as retention coefficients, learning style parameters, social, motivational and emotional factors, and parameters related to teaching strategies. These parameters could be incorporated into differential equations or stochastic processes describing the evolution of students' knowledge and individual characteristics over time.
Next, learning activities and teacher feedback could be modelled in a similar way to the first version of the model,

but taking into account individual student characteristics and their evolution over time.
Here is the mathematical model using differential equations:

- Modelling the evolution of students' knowledge:
  Let $e_{ij}(t)$ be the probability that student e_i has mastered skill sij at time t. We can define a differential equation for each skill S_{ij} as follows:

$$\frac{de_{ij}}{dt} = f(e_{ij}, t, activities, teacher\ feedback, individual\ parameters)$$

  where f is a function that describes how the probability of mastering the skill changes over time as a function of learning activities, teacher feedback and individual student parameters.
- Modelling learning activities and teacher feedback:
  When a student takes part in an Aj activity, he may make errors depending on his current knowledge and the requirements of the activity. The probability of error pij(t) for skill sij at time t can be modelled as a function of the similarity between the student's skills and the skills required for the activity :

$$p_{ij}(t) = g(e_{ij}, t, R_j, individual\ parameters)$$

  Teacher feedback can be modelled by adjusting student knowledge based on mistakes made and feedback provided. For example, individual student parameters could be adjusted according to the nature and effectiveness of the feedback.
- Modelling individual parameters :
  Individual parameters that influence student learning can also change over time. For example, a student's retention capacity could be modelled by a differential equation that describes how this capacity changes as a function of the student's learning experience.

$$\frac{dr_i}{dt} = h(r_i, t, activities, teacher\ feedback)$$

  By combining these elements, we can build a comprehensive mathematical model that captures student interactions, learning activities, teacher feedback and the various factors that influence the learning process. This model can be used to simulate and analyze various classroom learning scenarios and to inform the design of more effective teaching strategies.

4. The four scenario

In order to extend the model to a population of 50 learners, it is necessary to generalize the differential equations so as to represent the collective dynamics of the group. This theoretical framework aims to formalize the learning process within a cohort of 50 students.

4.1. Modelling the evolution of students' knowledge

We now have 50 students in the class, so for each skill S_{ij}, we will have a probability $e_{ij}(t)$ for each of the 50 students. So the system of differential equations becomes :

$$\frac{v}{e_{ij}^k} t = f(e_{ij}^k, t, activities, teacher\ feedback, individual\ parameters)$$

where k varies from 1 to 50 to represent each student in the class.

4.2. Modelling learning activities and teacher feedback:

The probability of error $p_{ij}^k(t)$ for each student k and each skill S_{ij} at time t can be calculated individually for each student based on their own knowledge and the requirements of the activity.

4.3. Modelling individual parameters

Individual parameters such as retention capacity, learning style preferences, social, motivational and emotional factors can also be modeled for each student in the class.
By applying this model to a class of 50 students, we can simulate how students' knowledge evolves over time in response to learning activities and teacher feedback, taking into account individual differences between students. This simulation could provide valuable information about how different factors influence learning in a classroom and could help to design more effective teaching strategies.

5. Modelling the evolution of students' knowledge

To illustrate how the model works, let's take arbitrary numerical values for the parameters and functions, then plot the curves for a few students in the class. Here is a simplified example with a single skill and a single student for visualisation purposes:
Suppose we have one skill S_{ij} and only one student in the class. The differential equations could be simplified as follows

$$\frac{de_{ij}}{dt} = -\alpha e_{ij} + \beta$$

Where α is a decay coefficient representing the natural forgetting of knowledge over time, and β is a term representing constant learning by the student.

5.1. Initialisation

We can initialise the student's knowledge at a certain level, for example $e_{ij}(0) = 0.2$, which means that the student initially masters the skill at 20%.

5.2. Solutions of differential equations

By solving these differential equations numerically, we can obtain the values of e_{ij} over time for this student.
We can also introduce a learning activity at some point, where the student is exposed to the skill sij and makes errors. This can be represented by a temporary drop in the probability of mastering the skill.

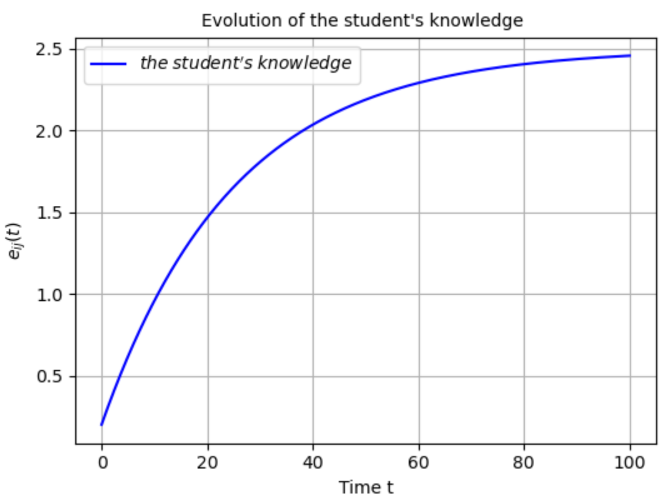

Figure 1. Evolution of the student's knowledge

Graph1 showing how the probability of mastering the skill changes over time for a given student in the class. We adjust parameter values and explore different situations to better understand how the model works.

5.3. *Exploration of three different situations*

Let's explore three different situations by adjusting the parameter values to see how they affect the evolution of the student's knowledge over time.
Situation 1: Low forgetting rate, high learning rate

- Parameter:
  - Knowledge decay rate $\alpha = 0.05$
  - Constant learning rate $\beta = 0, 4$

Situation 2: High forgetting rate, low learning rate

- Parameter:
  - Knowledge decay rate $\alpha = 0.2$
  - Constant learning rate $\beta = 0, 1$

Situation 3: Moderate forgetting rate, moderate learning rate

- Parameter:
  - Knowledge decay rate $\alpha = 0.1$
  - Constant learning rate $\beta = 0, 2$

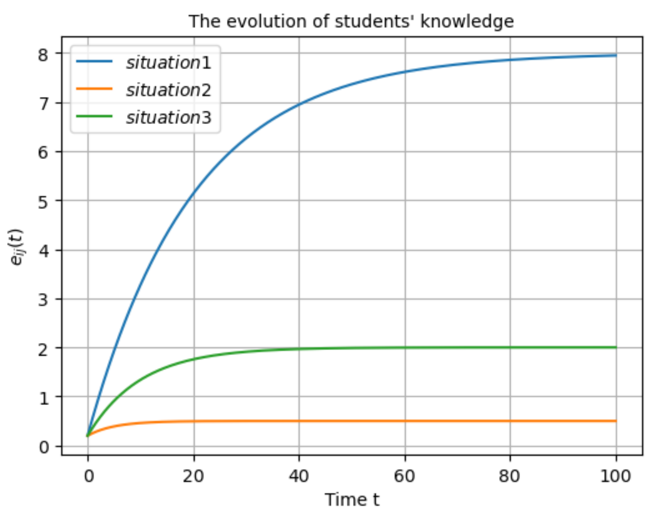

Figure 2. Evolution of student's knowledge

Let's analyse the overall interpretation of the curves 2 and then interpret each curve individually:

Global interpretation

- High forgetting and learning rates (Situation 1)
  - The probability of mastering the skill increases in 2 rapidly and remains high over time.

  - This suggests that despite a low forgetting rate, the high learning rate enables the student to rapidly acquire and maintain a good command of the skill.
- High forgetting rate and low learning rate (Situation 2) :
  - The probability of mastering the skill in 2 decreases rapidly and remains low over time.
  - This indicates that despite initial knowledge, a high rate of forgetting leads to a rapid loss of mastery, and a low rate of learning does not compensate for this loss.
- Moderate forgetting and learning rates (Situation 3) :
  - The probability of mastering the skill in 2 increases initially but tends to stabilise at an intermediate level.
  - This suggests that with moderate rates of forgetting and learning, the student acquires reasonable mastery of the skill but fails to maintain complete mastery in the long term.

Interpretation of each curve

- Situation 1 (low forgetting rate, high learning rate)
  - The student quickly acquires a high level of mastery of the skill and maintains it over time thanks to a low rate of forgetting and a high rate of learning.
- Situation 2 (high forgetting rate, low learning rate)
  - The student quickly loses mastery of the skill due to a high rate of forgetting, and a low rate of learning does not compensate for this loss.
- Situation 3 (Moderate forgetting and learning rates)
  - The student acquires a reasonable mastery of the skill, but gradually loses it due to a moderate rate of forgetting, and a moderate rate of learning does not allow high mastery to be maintained over the long term.

In summary, the different combinations of forgetting and learning rates have a significant impact on the development of student knowledge over time, underlining the importance of striking a balance between these factors to promote effective and sustainable learning.

Teacher intervention depending on the situation

In each of these situations, communication and collaboration with students become essential elements in understanding their individual needs and the challenges they face in their learning process. Teachers can encourage open dialogue by providing regular opportunities for students to share their experiences, questions and concerns. By establishing a relationship of trust and demonstrating a willingness to respond to individual needs, teachers can create an inclusive learning environment where students feel supported and motivated to meet academic challenges. What's more, by using data on student performance and carefully monitoring their progress, teachers can proactively adjust their teaching strategies to better meet the specific needs of each student and foster continued growth.
In each situation, teachers could adapt their interventions to better meet the needs of the students, based on the characteristics observed in the evolution of knowledge. Here's how they could intervene in each situation:

- Situation 1 in 2 (low forgetting rate, high learning rate): In this situation, where students rapidly acquire a high level of mastery of the skill and maintain it over time, teacher interventions could focus on :
  - Enrichment : Suggest activities that go beyond basic knowledge to challenge students who have already acquired a high level of mastery.
  - Individualised support: Identify students who may have specific needs or who may be bored by the speed with which they master the content, and provide them with additional challenges or research projects.

  - Formative assessment: Use formative assessments to quickly identify any gaps despite apparent strong mastery, and provide specific feedback to help students continue to make progress.
- Situation 2 in 2 (high forgetting rate, low learning rate):
  In this situation, where students quickly lose mastery of the skill due to a high rate of forgetting and where a low rate of learning does not compensate for this loss, teacher interventions could focus on :
  - Continuous reinforcement: Regular repetition of key concepts to help students consolidate their knowledge and prevent them from forgetting.
  - Differentiation: Offer differentiated activities that enable students to progress at their own pace and receive extra support where they need it.
  - Learning strategies: Teach students effective learning strategies to help them retain information better, such as note-taking, spaced repetition or creating summaries.
- Situation 3 in 2 (Moderate forgetting and learning rates):
  In this situation, where students acquire reasonable mastery of the skill but fail to maintain high mastery over the long term, teacher interventions could focus on:
  - Regular revision: Plan regular revision sessions to help students consolidate their knowledge and maintain their mastery over the long term.
  - Constructive feedback: Provide regular and constructive feedback to students on their performance, focusing on areas where improvement is needed to maintain mastery.
  - Encouraging autonomy: Encouraging students to take charge of their own learning by providing them with tools and resources so that they can continue to progress independently.

  By adapting their interventions to the specific characteristics of each situation, teachers can better support students in their learning and encourage continuous and sustainable progress.

6. Illustration of the mathematical model of learning activities and teacher feedback

To illustrate the mathematical model of learning activities and teacher feedback, let's take arbitrary numerical values for the parameters and plot the curves for a specific skill and student in the class. Here is an example with arbitrary values: Suppose we have a specific skill sij and a single student in the class. We are going to model the evolution of the probability of mastering this skill over time, taking into account learning activities and teacher feedback. This is how we might formulate the mathematical model:

- Learning activities:
  - Suppose the student is exposed to a learning activity at $t = 5$ time units, which increases the probability of mastering the skill.
  - The probability of mastering the skill after the activity can be modelled as an increasing function of the duration since the start of the activity.
- Teachers' feedback
  - Suppose that at t=10 time units, the teacher provides feedback that adjusts the probability of mastering the skill based on the errors made by the student.
  - Correcting errors could increase the probability of mastery of specific skills where errors have been made.

We can use an exponential function to model the evolution of the probability with an asymptote defined by

$$P(t) = P_0 + (P_{max} - P_0)(1 - e^{-\alpha t})$$

, where P_0 is the initial probability, P_{max} is the maximal probability, and α is the growth parameter.

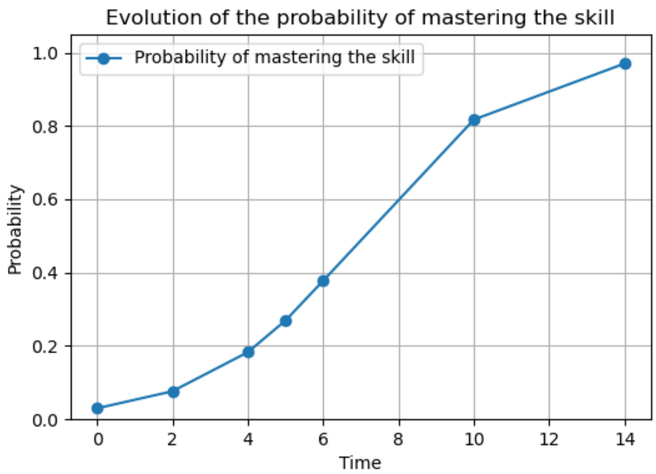

Figure 3. Evolution of the probability of masterinig the skill

Graph3 showing how the probability of mastering the skill changes over time for a given student in the class, taking into account learning activities and teacher feedback. You can adjust the learning activity and teacher feedback functions to reflect different learning and error correction scenarios.

6.1. Different learning and error correction scenarios

Let's explore four different situations, adjusting the learning activities and teacher feedback to illustrate different scenarios. Here's how we might formulate these situations:

- Situation 1: Progressive learning without errors
  - In this situation, the student progresses linearly in mastering the skill over time without making any mistakes. The learning activities are designed to be progressive, with no need for feedback.
    Then $P(t) = P_0 + rt$.
    where r is the rate of progression
- Situation 2: Learning with error correction.
  - In this situation, the student makes initial progress but makes a mistake at a certain point. The teacher then provides feedback that corrects this error, allowing the student to recover and continue to make progress.
    Then: $P(t) = P_0 + rt - ke^{-bt}$.
    where k is the corrected error impact factor, e^{-bt} the function representing errors committed
- Situation 3: Learning with uncorrected errors
  - In this situation, the student makes initial progress but makes a mistake at a certain point. However, the error is not corrected by the teacher, resulting in stagnation or a decrease in the probability of mastering the skill.
    Then: $P(t) = P_0 + rt - \alpha t$.
    where α is uncorrected error reduction coefficient
- Situation 4: Learning with errors and multiple correction
  - In this situation, the student makes several mistakes during the learning process. The teacher provides feedback for each mistake, enabling the student to correct his shortcomings and continue to make progress despite the difficulties encountered.
    Then: $P(t) = P_0 + \sum_{i=1}^{n} \Delta P.f(t - t_i)$.

where n is total number of corrections, ΔP is the increase in probability after each correction, and f is the Heaviside function or the Dirac function.

We will now plot the curves for each situation using different functions to model the learning activities and teacher feedback.

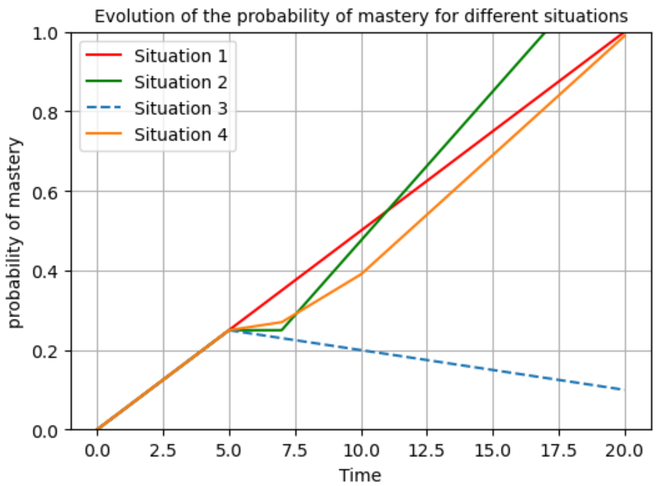

Figure 4. Evolution of the probability of mastery for different situations

Let's analyse the overall interpretation of the curves 4 and then interpret each curve individually:

6.2. *Global interpretation*

- The curves 4 represent the evolution of the probability of mastery of a specific skill over time in different learning situations and teacher feedback.
- Each curve in 4 illustrates how the probability of mastery changes in response to learning activities and teacher feedback, providing an overview of the student's progress in acquiring the skill.

6.3. *Interpretation of each curve*

- Situation 1: Progressive learning without errors
  - The probability of mastering the skill in 4 increases linearly over time, suggesting that the student is making steady progress in acquiring the skill without making any mistakes.
  - This situation could correspond to continuous and effective learning without major obstacles.
- Situation 2: Learning with error correction
  - The probability of mastering the skill in 4 increases gradually, but there is a sudden increase after a certain point, indicating that the teacher has corrected an error.
  - This situation suggests that an error has been made by the student, but that it has been identified and corrected, allowing the student to continue to progress.
- Situation 3: Learning with uncorrected errors
  - The probability of mastering the skill in 4 increases initially, but decreases after a certain point, indicating that an error has been made but not corrected.
  - This situation highlights the importance of teacher feedback in correcting errors and maintaining student progress.

- Situation 4: Learning with errors and multiple correction
  - The probability of mastering the skill in 4 shows fluctuations with several increases and decreases, indicating multiple errors and corrections over time.
  - This situation highlights the need for ongoing, tailored feedback to support the student in correcting shortcomings and making further progress.

In summary, each curve in 4 represents a different learning and teacher feedback scenario, highlighting the impact of these factors on the student's progress in acquiring a specific skill.

6.4. *Teacher intervention depending on the situation*

Let's explore how teachers can intervene in each situation to support students in their learning:

- Situation 1: Progressive learning without errors.
  In this situation, where the pupil progresses in a linear fashion without making any mistakes, teachers could:
  - Encourage and value the student's steady progress to maintain motivation.
  - Provide additional activities or challenges to maintain engagement and stimulate curiosity.
- Situation 2: Learning with error correction.
  When a student makes a mistake but receives a correction from the teacher, the teacher could :
  - Identify and clearly explain the error made by the student, providing examples and additional explanations if necessary.
  - Encourage the student to understand the error and actively correct it, providing additional opportunities for practice.
- Situation 3: Learning with uncorrected error.
  If a student makes a mistake but it is not corrected, teachers may :
  - Quickly identify the student's shortcomings and provide specific feedback to help correct the error.
  - Organise individual or group tutoring sessions to help students overcome their difficulties and regain confidence in their abilities.
- Situation 4: Learning with errors and multiple correction.
  When students make several mistakes and receive multiple corrections, teachers could :
  - Adopt a differentiated approach by identifying the student's specific needs and providing personalised support for each identified gap.
  - Encourage students' perseverance and resilience by stressing the importance of learning through mistakes and continuous correction.

In all situations, teachers play a crucial role in providing support tailored to students' individual needs, identifying and correcting errors constructively, and encouraging continuous progress in the acquisition of skills. Open communication and a relationship of trust with students are essential to fostering a positive and productive learning environment.

7. Learning by stages

Let's explore another type of curve that could represent the evolution of the probability of mastery of a specific skill over time. We will use an approach based on learning by stages, where the probability of mastery increases significantly after each teacher intervention or intensive learning session.
Here's how we might formulate this approach:

- Initially, the probability of mastering the skill is low.

- After each teacher intervention or intensive learning session, the probability of mastery increases significantly.
- Between these interventions, the probability of mastery may remain relatively stable or decrease slightly, reflecting the effect of forgetting or other factors.

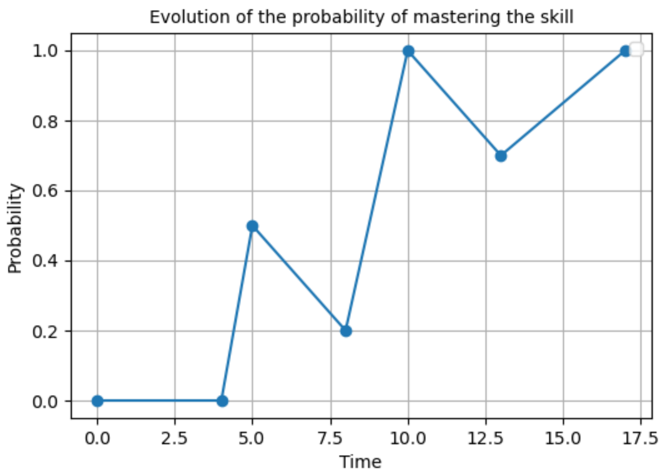

Figure 5. Evolution of the probability of masterning the skill

Graph 5 showing the evolution of the probability of mastering the skill over time, using an approach based on learning levels. Each level represents a teacher intervention or intensive learning session that leads to a significant increase in the probability of mastering the skill.

7.1. Exploration of four different situations

Let's explore four different situations by modelling the evolution of the probability of mastering a specific skill over time. For each situation, we will use different approaches to represent learning activities and teacher feedback. Here are the four situations we will explore:

- Situation 1: Constant progress without error.
  In this situation, the student makes steady progress without making any mistakes. Feedback from teachers is not necessary.
- Situation 2: Learning with errors and correction.
  In this situation, the student makes progress but makes a mistake at a certain point. The teacher then provides feedback to correct the error.
- Situation 3: Stagnant progress with feedback.
  In this situation, the student stagnates in his/her learning. The teacher intervenes by providing feedback to encourage progress.
- Situation 4: Erratic progress with multiple feedback.
  In this situation, the student makes erratic progress, making mistakes at different times. The teacher intervenes several times to provide feedback and correct the errors.

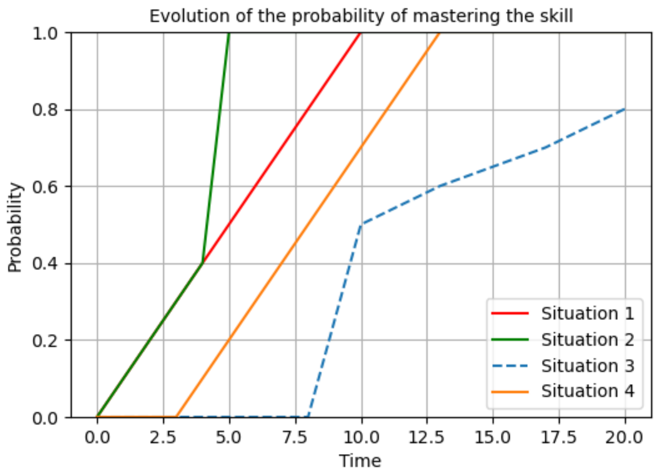

Figure 6. Evolution of the probabilty of mastering the skill

Graph 6 showing the evolution of the probability of mastering the skill over time for each situation. Let's interpret each curve individually, and then draw an overall interpretation for all the situations:

7.2. *Interpretation of each curve*

- Situation 1: Constant progress without error.
  - This curve 6 shows a linear progression in the probability of mastering the skill over time.
  - The absence of errors and feedback from teachers suggests that the student is making steady progress in acquiring the skill.
- Situation 2: Learning with errors and correction.
  - The probability of mastery in 6 initially increases steadily, but there is a sudden increase after a certain point, indicating a correction of error by the teacher.
  - This situation suggests that an error has been made by the student, but that it has been identified and corrected, allowing the student to continue to progress.
- Situation 3: Stagnant progress with return.
  - The probability of mastery in 6 increases slowly up to a certain point, then increases more rapidly after the teacher's intervention.
  - This suggests that the student has encountered difficulties or obstacles in his/her learning, but that the teacher's intervention has stimulated progress.
- Situation 4: Erratic progress with multiple returns.
  - This curve 6 shows fluctuations in the probability of mastering the skill, with increases and decreases at different times.
  - Multiple feedback from teachers seems to have a positive effect on student progress despite the difficulties encountered.

7.3. *Global interpretation*

- Overall, these curves illustrate different learning dynamics and teacher intervention.
- The absence of errors and teacher feedback (Situation 1) leads to steady progress, while error correction (Situation 2) and teacher feedback (Situation 3 and 4) stimulate progress in different ways.

- The situations where teacher feedback is provided (Situation 2, 3 and 4) show a significant increase in the probability of mastering the skill after teacher intervention, highlighting the importance of feedback in the learning process.
- The different responses for each curve highlight the crucial impact of student-teacher interactions on learning progression and underline the importance of appropriate pedagogical intervention to support student success.

7.4. Conclusion

In conclusion, the exploration of these different learning and teacher intervention situations highlights the importance of the dynamic between students and teachers in the skills acquisition process. Each situation illustrates a unique facet of this relationship, highlighting both the challenges faced by students and the crucial role of teachers in supporting them and their progress.
The absence of errors and teacher feedback leads to steady progress, while error correction and teacher feedback stimulate progress in different ways. These tailored teaching interventions, whether aimed at correcting specific errors or encouraging general progress, are essential to guiding students along the path to mastery of skills.
By understanding the different dynamics illustrated by these situations, teachers can adjust their pedagogical approaches to meet students' individual needs and foster an inclusive and productive learning environment. By encouraging collaboration, offering personalised support and providing constructive feedback, teachers can play a decisive role in students' success and prepare them to meet future challenges.

8. Modelling curves for individual parameters

To draw the individual parameter modelling curves, we first need to define the specific parameters we want to model. For example, we might choose to model the progression of mastery of a specific skill over time for a group of students. Let's say we have a group of 50 students and a particular skill they are trying to master. We can model the probability of each student mastering this skill over time using a logistic growth function.

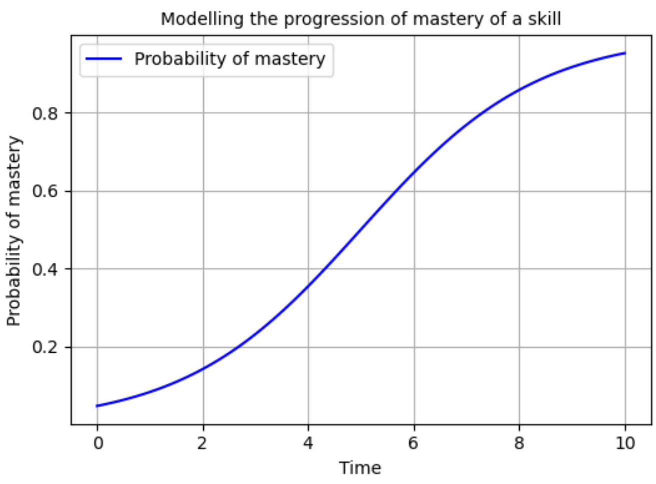

Figure 7. Modelling the progression of mastery of a skill

The curve 7 models the progression of the probability of a student mastering a specific skill over time, based on parameters such as the learning rate, the growth factor and the upper limit of the skill. You can adjust these parameters as required to explore different learning situations.

8.1. Different learning situations

To plot the modelling curves for individual parameters in different learning situations, we first need to define the specific scenarios we want to explore. Here are four different learning situations we might consider:

- Situation 1: Learning without teacher intervention, where students learn autonomously.
- Situation 2: Regular teacher intervention, where teachers provide frequent and targeted feedback to students.
- Situation 3: Adaptive learning, where learning resources are adapted to students' individual needs.
- Situation 4: Peer collaboration, where students work together to achieve common learning goals.

For each situation, we can model the progression of mastery of a specific skill over time for a group of students using a logistic growth function, while adjusting the parameters to reflect the unique characteristics of each situation.
the differential equation used to plot the curves is

$$\frac{dP}{dt} = rP(1-P),$$

where P is the probabilty of mastery, r is the growth rate.
The modelling curves in these four learning situations:

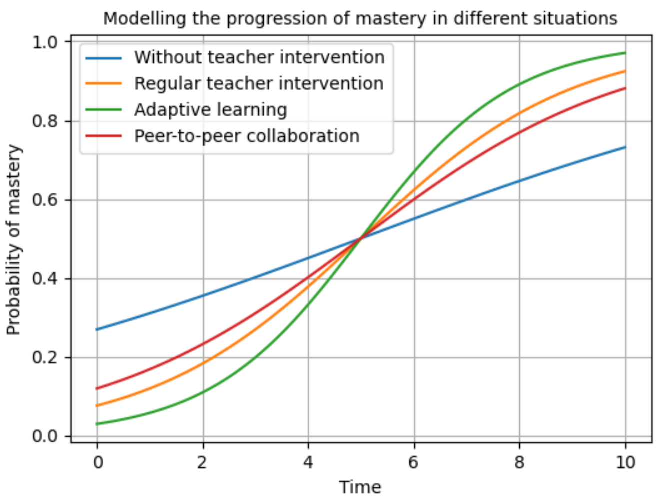

Figure 8. Modelling the progression of mastery in different situations

Graph 8 with four curves representing the progression of the probability of a group of students mastering a specific skill over time in each of the four learning situations. The alpha and beta parameters are adjusted for each situation to reflect the unique characteristics of each scenario.

8.2. Interpretation of each curve

- Without teacher intervention:
  The curve 8 shows a slow progression in the probability of mastering the skill over time, with a low learning rate (alpha) and a medium growth factor (beta). This suggests that without direct teacher intervention, students progress at a relatively slow pace in acquiring the skill.
- Regular teacher intervention:
  This curve 8 shows a faster increase in the probability of mastering the skill compared with the situation without teacher intervention. The higher learning rate (alpha) indicates that frequent, targeted feedback from teachers accelerates the learning process for students.

- Adaptive learning:
  In this situation, the curve 8 shows an even faster progression in the probability of mastering the skill. The high learning rate (alpha) combined with a medium growth factor (beta) suggests that adapting learning resources to the individual needs of students leads to significant gains in skill acquisition.
- Peer collaboration: The curve 8 shows a progression similar to that of regular teacher intervention, with an average learning rate (alpha) and an average growth factor (beta). This suggests that peer collaboration can provide effective support for learning, even without direct teacher intervention.

8.3. General interpretation

In general, these curves 1, 2, 3, 4, 5, 6, 7, 8 illustrate how different pedagogical approaches influence the progression of students' mastery of skills. Regular teacher intervention and adaptive learning seem to lead to greater gains in skill acquisition than situations without teacher intervention or based on peer collaboration. This highlights the importance of teacher-student interactions and of adapting teaching practices to the individual needs of learners.
individual needs of learners to promote effective learning. Furthermore, these results highlight the potential of modelling models to inform our understanding of learning processes and guide the development of evidence-based pedagogical practices.

9. General conclusion

In conclusion, this study has enabled us to explore different learning situations by modelling individual parameters. By examining the progression of skills mastery in a variety of contexts, we have been able to identify the factors that influence students' learning processes.
Our results underline the crucial importance of teacher-pupil interactions and the adaptation of teaching practices to promote effective learning. Regular teacher interventions and adaptive learning proved particularly effective in stimulating student progress, while peer collaboration also showed promising results.
This study highlights the potential of modelling models to inform our understanding of learning processes and guide the development of evidence-based teaching practices. By combining sound theoretical approaches with advanced modelling methods, we can better understand how interactions between teachers, students and learning content shape the educational experience.
Moving forward, further research is needed to refine these models and explore other aspects of educational practice. By continuing to use modelling as a tool to study learning dynamics, we can contribute to the continuous improvement of education.

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