



The Prediction by using Nonlinear Autoregressive Model

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Abstract The study proposed a nonlinear autoregressive model along with its stability conditions. The primary objective was to apply this model to predict the daily number of new COVID-19 cases in the Kingdom of Saudi Arabia during 2022. The model consists of two parts: a linear term and a nonlinear term that incorporates a decreasing function. This structure made it possible to construct a numerical example that meets the theoretical stability criteria, as illustrated in Example 1 of the paper. The model was then applied to real-world data, specifically the time series of daily COVID-19 infections recorded over a continuous three-month period in 2022. Using a Python-based implementation, the model parameters were estimated to fulfill the stability conditions. The residuals were analyzed, and the model was subsequently employed to forecast the expected number of new infections for the following month of 2022.

Keywords Limit cycle; Singular point; coefficient estimation; prediction; stability analysis.

AMS 2010 subject classifications 62M10, 62M20, 62M99, 62P99

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1. Introduction

A nonlinear autoregressive model was proposed that simulates the exponential autoregressive model of the scientist Ozaki and we used the "local linearization technique" to obtain an approximate linear model for the proposed study model. We first determined the single point of the proposed model, then determined the stability criteria of this single point and the stability of the limit cycle. To illustrate this technique, we mentioned examples 1 and 2, to illustrate this technique. The research model was used to analyze the real time series data related to COVID-19 infection in the Kingdom of Saudi Arabia during a period of three months in 2022 from the website <https://www.worldometers.info/coronavirus/country/saudi-arabia/>. To ensure the stability of the time series data, we applied the natural logarithm transformation because the original data is increasing exponentially, and then the first difference was taken to stabilize the time series as shown in Figure 7. For the research, Python programs were employed to process the data and estimate the parameters of the proposed first-order nonlinear model that met the stability criteria. Finally, we applied the study model to predict future COVID-19 cases in the Kingdom of Saudi Arabia, which is the goal of the research.

We can see the definition of the ARMA model of order (p, q) in [1]. Definitions of the "autoregressive model of time series of order p in discrete time," the "exponential nonlinear model of order p ," the "singular point with its stability," the "limit cycle with its stability," and the technique of "local linearization" were given in [2]. Also, the "exponential nonlinear model with γ of order p " is defined in [3].

Numerous scholars have identified the criteria for ensuring the stability of nonlinear regression models through the utilization of the linear local approximation technique pioneered by Ozaki. The stability conditions for a

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limit cycle in the Gompertz autoregressive model were investigated in [4]. The stationary condition for the GJR-GARCH(Q, P) model was determined in [5]. Stability conditions for SATER models were found in [6], and for nonlinear time series models in [7]. Stability conditions for exponential GARCH models were studied in [8]. The stability of boiling water reactor oscillation through nonlinear time series modelling was studied in [9]. The stability analysis of RBF network-based state-dependent autoregressive models for nonlinear time series was conducted in [10].

Many scholars have employed regression models to forecast future values of analyzed data. Maobin Li, Shouwen Ji, and Gang Liu used three forecast models for ECS in [11]. Models were developed to forecast wind speeds over timescales ranging from 10 minutes to one hour in [12]. Forecasts for COVID-19 using a simple frequency method based on daily confirmed cases were generated in [13]. The future of the COVID-19 pandemic in Europe in 2021 was studied in [14]. Data from cohorts vaccinated against COVID-19 and BCG were examined to analyze how both COVID-19 and TB spread using a mathematical partitioned model in [15].

A new nonlinear autoregressive neural network time series model (NAR-NNTS) was introduced for predicting COVID-19 cases in [16]. A univariate time series model was used to predict the number of COVID-19 cases likely to occur in the coming days in India in [17]. A time series model using a nonlinear regression extrinsic neural network (NARX) was proposed for predicting recovered and deceased COVID-19 cases in [18]. The nonlinear smoothed autoregressive transition (STAR) model was demonstrated to improve the prediction of COVID-19 incidence rate in [19]. Linear and nonlinear forecasting models were confirmed to accurately capture the COVID-19 trend in Nigeria in [20].

Models were used to predict COVID-19 data using improved LSTM-ARIMA algorithms in [21]. A hybrid approach combining ARIMA and a single hidden layer autoregressive neural network (NNAR) was used to forecast daily COVID-19 cases in [22]. The accuracy of the best-fit ARIMA model forecasts was evaluated in [23]. A comparison between ARIMA and XGBoost models for predicting COVID-19 in the United States was conducted in [24]. COVID-19 confirmed cases and deaths in Chile were predicted using time series techniques in [25].

A deep belief network-based state-dependent autoregressive (DBN-AR) model was investigated in [26]. A newly developed nonlinear autoregressive model was applied to forecast daily new COVID-19 cases in Iraq in [27]. The NARNN framework employs the sigmoid activation function under varying settings of previous values and hidden neuron counts in [28]. A nonlinear autoregressive exogenous (NARX) model was constructed to forecast the spread of COVID-19 in Jordan in [29]. The study investigates the appropriateness of applying the K-means clustering algorithm to categorize five separate datasets that undergo feature selection through the Binary Harris Hawks Optimization Algorithm (BHHA) in [30]. The population of Egypt for the period 2024–2033 was predicted using a non-linear model proposed in [31].

2. THE PROPOSED STUDY FRAMEWORK

The proposed model for studying nonlinear time series is

$$y_t = \sum_{i=1}^p \left[a_i + b_i \left(\frac{1}{2y_{t-1}} \right) \right] y_{t-i} + \varepsilon_t, \quad y_{t-1} \neq 0 \quad (1)$$

where, $\{\varepsilon_t\}$ is white noise $a_1, \dots, a_p; b_1, \dots, b_p$ are the constants of equation (1).

If $y_{t-1} \rightarrow \mp\infty$, then

$$\frac{1}{y_{t-1}} \rightarrow 0,$$

the model of equation (1) is $y_t = \sum_{i=1}^p a_i y_{t-i} + \varepsilon_t$

3. RESULTS

The technique employed to examine the stability of the suggested model is an approximation method.

3.1. Singular point Z

Replacing with $p=1$ in equation (1) to get

$$y_t = \left[a_1 + b_1 \left(\frac{1}{2y_{t-1}} \right) \right] y_{t-1} + \varepsilon_t, \quad y_{t-1} \neq 0 \quad (2)$$

Since $\varepsilon_t = 0$, $Z = f(Z)$. Then

$$Z = \left[a_1 + b_1 \left(\frac{1}{2Z} \right) \right] Z, \quad Z \neq 0, b_1 \neq 0 \quad (3)$$

The singular point Z

$$Z = \frac{b_1}{2(1 - (a_1))} \quad (4)$$

3.2. Stability at a singular point:

The stability conditions for a singular point were formulated using $y_s = Z + Z_s, \forall s = t; t - 1$, and for an equation (2), $\varepsilon_t = 0$, and since, $\forall s = t; t - 1; Z_s$ is smallest, to reached that Z_s^n convergent to zero, $\forall n \geq 2, \forall s = t; t - 1; Z_t \cdot Z_{t-1} = 0$.

$$Z + Z_t = \left[a_1 + b_1 \left(\frac{1}{2(Z + Z_{t-1})} \right) \right] (Z + Z_{t-1}) \quad (5)$$

Then,

$$Z_t = \left[\left(\frac{4a_1 Z + b_1 - 2Z}{2Z} \right) \right] Z_{t-1} = m_1 Z_{t-1}, \quad m_1 = \left[\left(\frac{4a_1 Z + b_1 - 2Z}{2Z} \right) \right] \quad (6)$$

If the solution to equation (6) is within a unit circle, then (6) represents a stable first-order linear model. Expressed using symbols

$$|n_1| = |m_1| < 1$$

3.3. A limit cycle

A period q limit cycle of $y_t = y_t; y_{t+1}; \dots; y_{t+q}$; for the model suggested in Equation (2). When y_s is a points nearly a limit cycle is replaced

$$\forall s \in \{t, t - 1\}, \quad y_s = y_s + Z_s$$

$$y_t + Z_t = \left[a_1 + b_1 \left(\frac{1}{2(y_{t-1} + Z_{t-1})} \right) \right] (y_{t-1} + Z_{t-1}) \quad (7)$$

Then

$$Z_t = \left(\frac{4a_1 y_{t-1} + b_1 - 2y_t}{2y_{t-1}} \right) Z_{t-1} \quad (8)$$

Put, $t = t + q$ in (8) forgetting

$$Z_{t+q} = \left(\frac{4a_1 y_{t+q-1} + b_1 - 2y_{t+q}}{2y_{t+q-1}} \right) Z_{t+q-1} \quad (9)$$

Therefore

$$Z_{t+q} = \prod_{i=1}^q \left[\frac{4a_1 y_{t+q-i} + b_1 - 2y_{t+q-(i-1)}}{2y_{t+q-i}} \right] Z_t \quad (10)$$

Then

$$\left| \frac{Z_{t+q}}{Z_t} \right| = \left| \prod_{i=1}^q \left(\frac{4a_1 y_{t+q-i} + b_1 - 2 y_{t+q-(i-1)}}{2 y_{t+q-i}} \right) \right| < 1 \tag{11}$$

Then, (11) is

$$\left| \frac{Z_{t+q}}{Z_t} \right| = \left| \prod_{i=1}^q \left(\frac{4a_1 y_{t+i-1} + b_1 - 2 y_{t+i}}{2 y_{t+i-1}} \right) \right| < 1 \tag{12}$$

4. Examples

This section of the study comprises two examples, denoted as examples (1) and (2). These illustrations serve the purpose of pinpointing the singular point for the proposed model, applying stability constraints, and charting the trajectories of first-order models. Additionally, you can refer to Appendix (A) for MATLAB programs that display the orbits of the study model for both Examples (1) and (2).

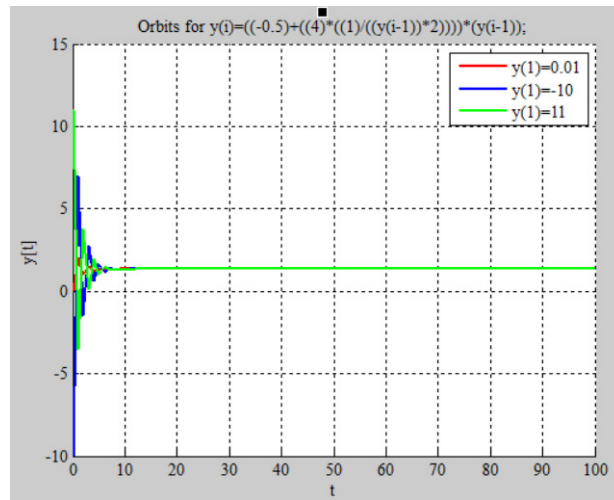


Figure 1. The stability of a singular point with different initial values.

4.1. Example (1)

Let's suppose that we have the given model. $y_t = [-0.5 + 4(\frac{1}{2y_{t-1}})]y_{t-1} + \varepsilon_t$ While $a_1 = -0.5, b_1 = 4$. By using equation (4), we derive that the singular point is such that:

$$Z = \frac{b_1}{2(1 - (a_1))} = \frac{4}{2(1 + 0.5)} = \frac{4}{2(1.5)} = 1.333$$

When $Z = 1.333$, and by using (6),

$$Z_t = -0.8878 Z_{t-1} \quad (*)$$

Therefore, $Z = 1.333$ is a stable because the root $n_1 = -0.8878$ of equation (*) is in a unit circle.

Therefore, the next figure display the stability of the proposed model with different initial values

$$y(1) = 0.01; y(1) = -10; y(1) = 11.$$

$Z = 1.333$, with constants $a_1 = -0.5, b_1 = 4$.

Figure 1, illustrates the trajectory plot for the stable proposed first-order model with a singular point

4.2. Example (2)

If , $a_1 = -1.9, b_1 = 4.2, y_t = [-1.9 + 4.2(\frac{1}{2^{y_{t-1}}})]y_{t-1} + \varepsilon_t$ By utilizing equation (4), we determine that the singular point is such that: $Z = \frac{b_1}{2(1-(a_1))} = \frac{4.2}{2(1+1.9)} = \frac{4.2}{2(2.9)} = 0.7241$ When $Z = 0.7241$ and using (6), we obtain

$$Z_t = -1.8998 Z_{t-1}.$$

Therefore, $Z = 0.7241$ is unstable (the root $n_1 = 1.8998$) is outside a unity circle.

Therefore, the figure shows model orbits that unstable in different initial values.

$$y(1) = 0.1; y(1) = -10; y(1) = 11;$$

Figure 2, illustrates the trajectory plot for unstable proposed first-order model with a singular point $Z = 0.7241$, with constants $a_1 = -1.9, b_1 = 4.2$.

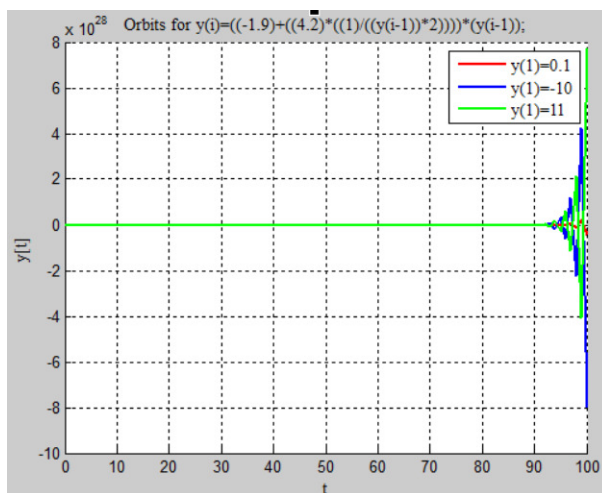


Figure 2. unstable singular point in different initial values.

5. Application

5.1. Description of the data utilized in the study

The study utilized information gathered from the website <https://www.worldometers.info/coronavirus/country/saudi-arabia/>, which provides information on the daily count of new Covid-19 cases in Kingdom of Saudi Arabia between 1st April 2022 and 30th June 2022. The dataset comprises 91 observations, representing the number of new cases reported each day during this specific timeframe. The dataset’s lowest value occurred on 04-Apr-2022, with 78 new cases reported, whereas the highest value was recorded on 21-Jun-2022, with 1232 new cases reported, and to access the data used in the study, please refer to Appendix (A). The next figure, figure 3, display The $COVID = y_t$ series data between 01-Apr-2022 and 30-Jun-22. figure 3, the daily count of new Covid-19 cases in Kingdom of Saudi Arabia between 1st April 2022 and 30th June 2022.

By utilizing the SPSS program to analyze the graph of the primary COVID data series as shown in Figure 3, it becomes evident that the data is exhibiting growth, following an exponential pattern, and displaying instability. Through the utilization of the EViews 9 software, an analysis was conducted on the initial COVID dataset (y_t) using autocorrelation and partial auto correlation functions as depicted in Figure 4. The observation made from this analysis was that the COVID variable lacks stability Figure 4 for the autocorrelation and partial autocorrelation functions for the real data series COVID.

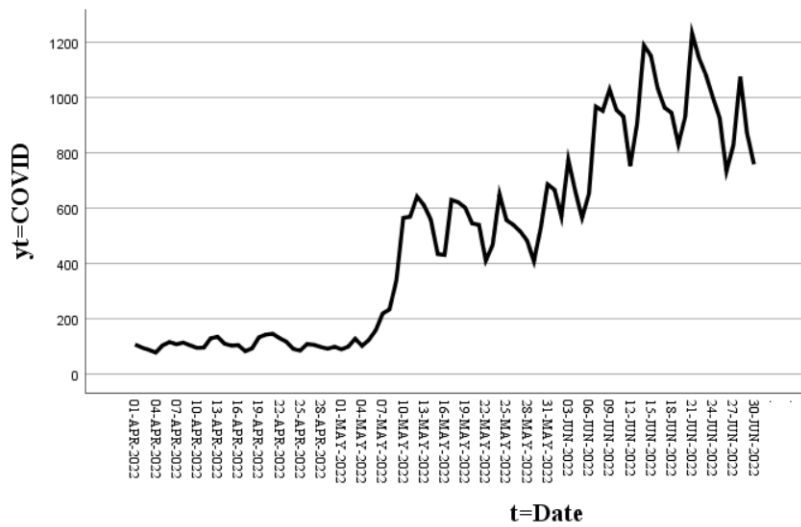


Figure 3. The COVID series data between 01-Apr-2022 and 30-Jun-22.

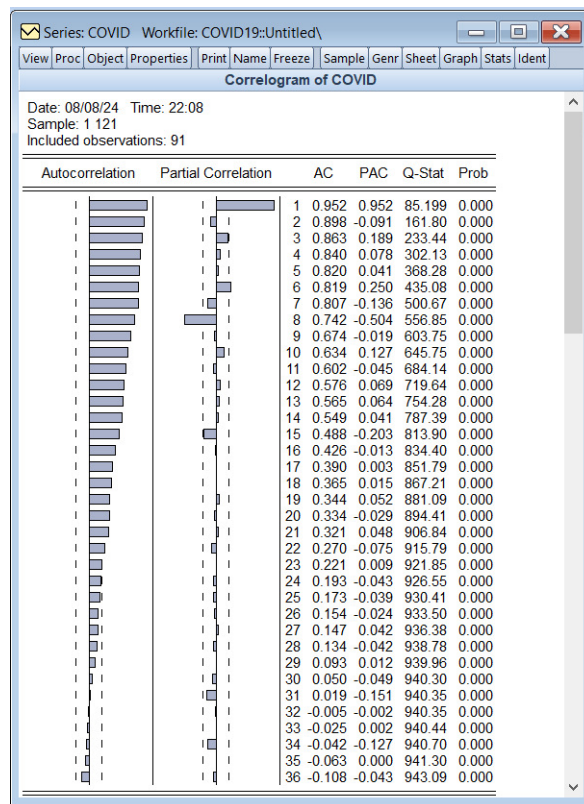


Figure 4. COVID’s autocorrelation and partial autocorrelation functions.

Due to the exponential nature of the original series, a fresh series named LCOVID was generated by logarithmically transforming the COVID data. Subsequently, autocorrelation and partial autocorrelation functions were graphed for the LCOVID series. Nonetheless, upon observation, the newly derived LCOVID time series

data was found to exhibit instability, as illustrated in Figure 5. Figure 5 for the autocorrelation and partial autocorrelation functions for the transformed data series LCOVID.

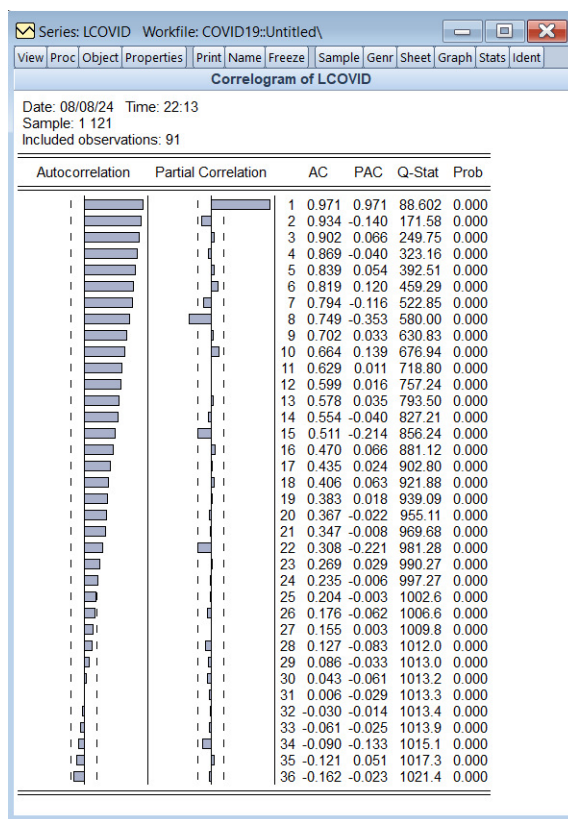


Figure 5. LCOVID Graphs of autocorrelation and partial autocorrelation functions.

After applying the first difference to the original data by taking the natural logarithm, we obtained a new variable,

$$DLCOVID = \log(y_t) - \log(y_{t-1}) = Z_t$$

The autocorrelation and partial autocorrelation functions were plotted for this new variable DLCOVID, as shown in Figure 6. Figure 6 for the autocorrelation and partial autocorrelation functions for the transformed data series with the first difference D(LCOVID).

The Figure 7, displays the resulting stable time series and can be used for predicting future observations. The following figure, figure 7, shows the stable series of logarithms of the first difference of the original data that $DLCOVID = Z_t$

5.2. Estimate the parameters and stability condition, forecasted, prediction for the study model with data

Programs in the Python language were used to estimate the values of the parameters suitable for the study data and the proposed non-linear model and to achieve the stability conditions of the research model, which were found on the theoretical side in the third paragraph of the research, which is related to the results. We can see the program in the python in the link:

https://colab.research.google.com/drive/1spI2QiMN0_6HjbZojfLLcwWtpts4RNk8B

5.2.1. Estimate the parameters for data and model with stability condition From the Python program we estimate the parameters for the nonlinear proposed model that suitable for the study data in our search. The program starts

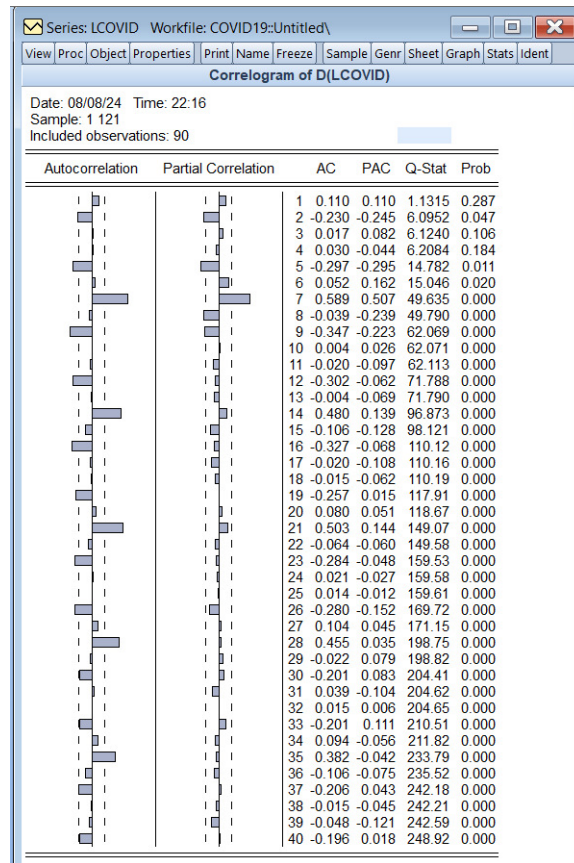


Figure 6. Graphs of autocorrelation and partial autocorrelation functions for LDCOVID= Z_t .

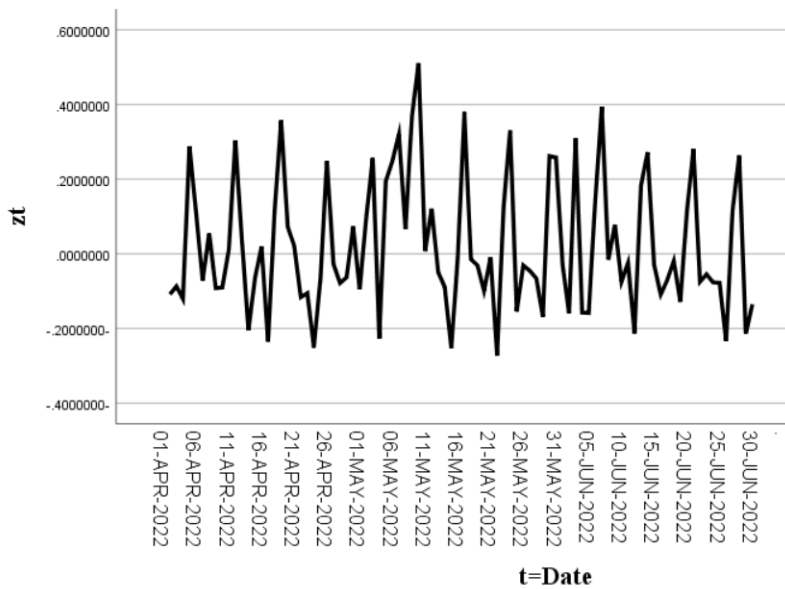


Figure 7. DLCOVID = Z_t , plot.

by employing random initial values for the estimated parameters. Then $c = a_1 = 0.97, d = b_1 = 0.27$ and by using equation (2), we get that:

$$y_t = [0.97 + 0.27(\frac{1}{2^{y_{t-1}}})]y_{t-1} + \varepsilon_t$$

By using equation (4), we get the singular point Z such that:

$$Z = \frac{b_1}{2(1-a_1)} = \frac{0.27}{2(1-0.97)} = \frac{0.27}{2(0.03)} = 4.5. \text{ When } Z = 4.5, \text{ and by using (6),}$$

$$Z_t = 0.97 Z_{t-1} \quad (**)$$

Therefore, $Z = 4.5$ is stable because the root $n_1 = 0.97$ of equation (**) is in a unit circle.

The next figure, figure 8 explain the stability singular point $Z = 4.5$, with estimate parameters $c = a_1 = 0.97, d = b_1 = 0.27$ that suitable the study data and study proposed model.

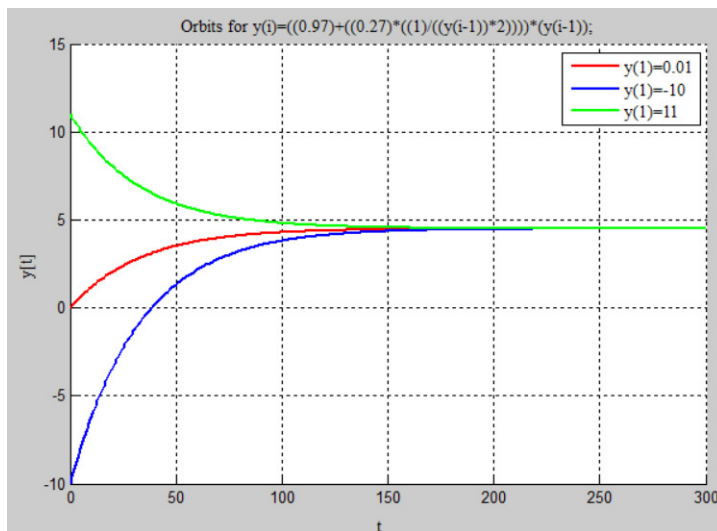


Figure 8. The stable proposed model that suitable the study data with estimated parameters.

5.2.2. *The forecasting for study model with transform data* We were forecasting by using the proposed model with the estimated parameters that we found in the Python program; we can see Appendix (B) to find the forecasting and residuals for the transformed data by using the study model and the statistical criteria were that:

The Mean Squared Error (MSE) = 0.031119718076058615, $ResidualVariance$ = 0.03181903758338577, AIC = 10.895381004361774, BIC = -304.718116685427, $NBIC$: -3.34855073280689, and the next figure, figure 9 show that the transform original data and forecasted data by using the study proposed model.

5.2.3. *The residuals test for the study model* The autocorrelation function and the partial autocorrelation function were found for the residuals of the model proposed in the first-order study for the transferred data, most of which fall within the limits of confidence, which indicates that the model agrees with the data used and the possibility of using it to predict future values. The next figures, figure 10 and figure 11 explained the autocorrelation and partial autocorrelation functions graphs for the study model residuals The residual is clearly white noise, indicating that the model is correct.

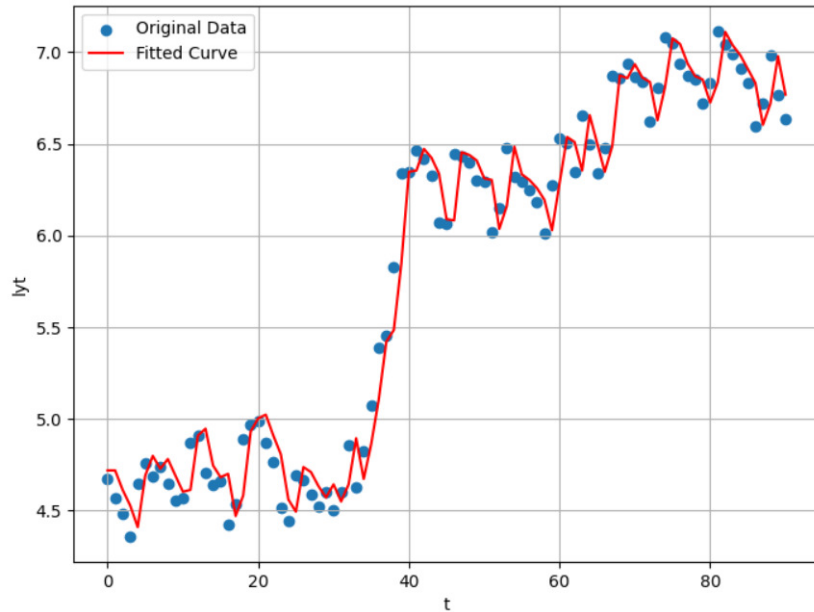


Figure 9. The forecasted data and transformed real data plot.

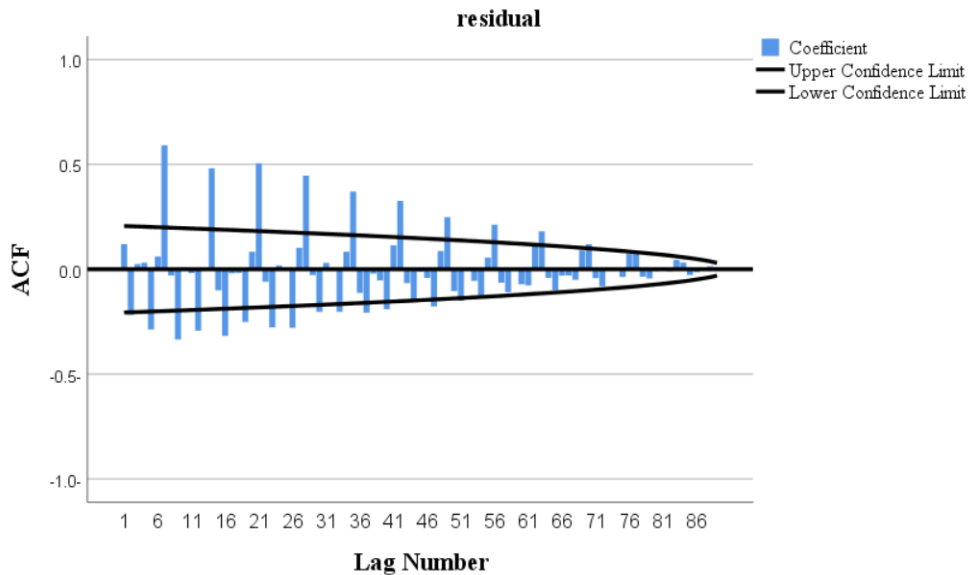


Figure 10. The autocorrelation function for the study model residuals.

5.2.4. We forecasted COVID data for the next month that we used by using study model We can see the program in the python in the link:

https://colab.research.google.com/drive/1udVKbUFF-0TjJ_Aycuq_yazRD1LZkzTf

From the Python program we estimate the parameters for the nonlinear proposed model that suitable for the study data of Covid in our search. The program starts by employing random initial values for the estimated parameters. Then, $c = a_1 = 0.959108457269065, d = b_1 = 53.09035963635171$ we can see in Appendix (C) that we found the

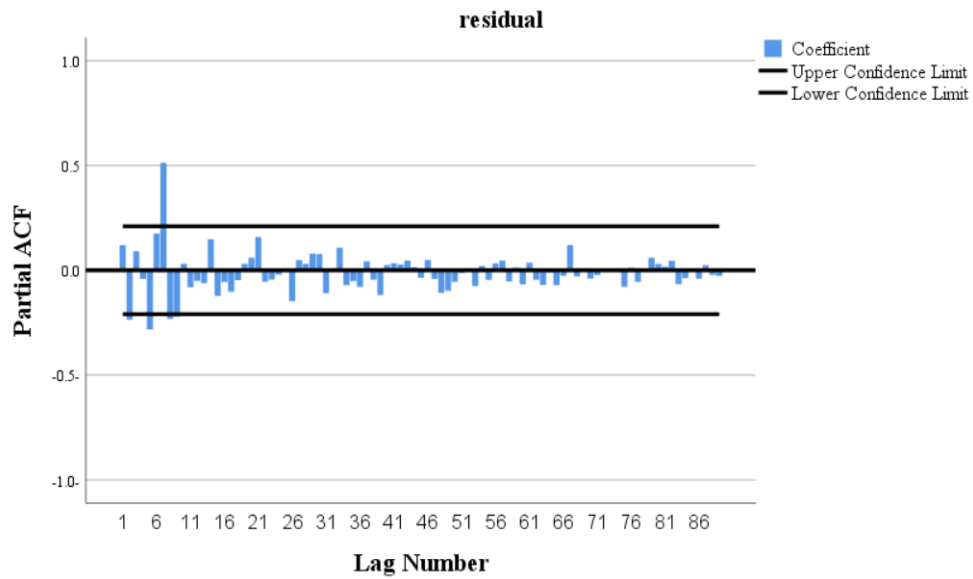


Figure 11. The partial autocorrelation function for the study model residuals.

forecasting COVID data by using the study model with the original data. The next figure, figure 12, shows the forecasting of COVID data by using the study model.

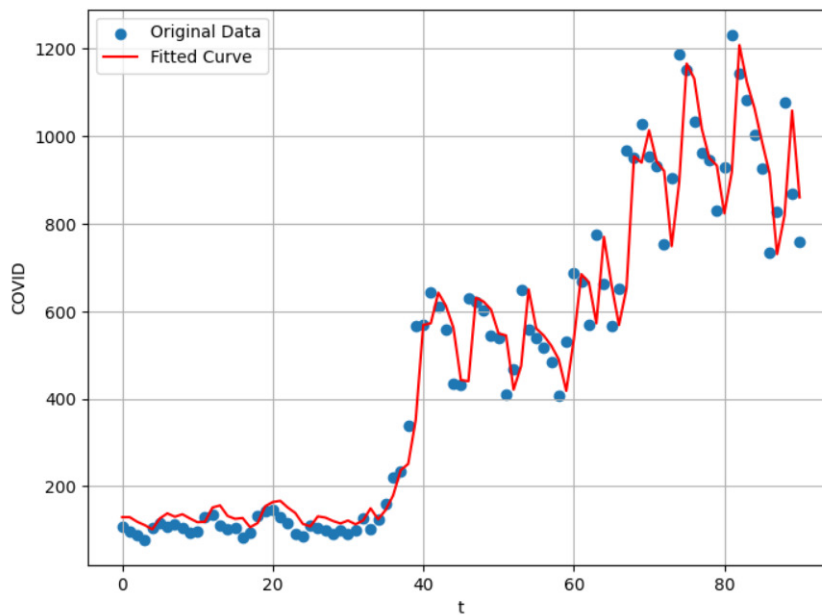


Figure 12. Forecasting COVID data by using the study model.

5.2.5. *The predictions for study model with real data* We can see the predictions for the study model with real data for the next thirty days. The next figure, figure 13, explain the prediction of the real COVID data and the real data

that we used in our search by using the study model for the next thirty days. Figure 13 the predictions for COVID data by using the study model for the next month.

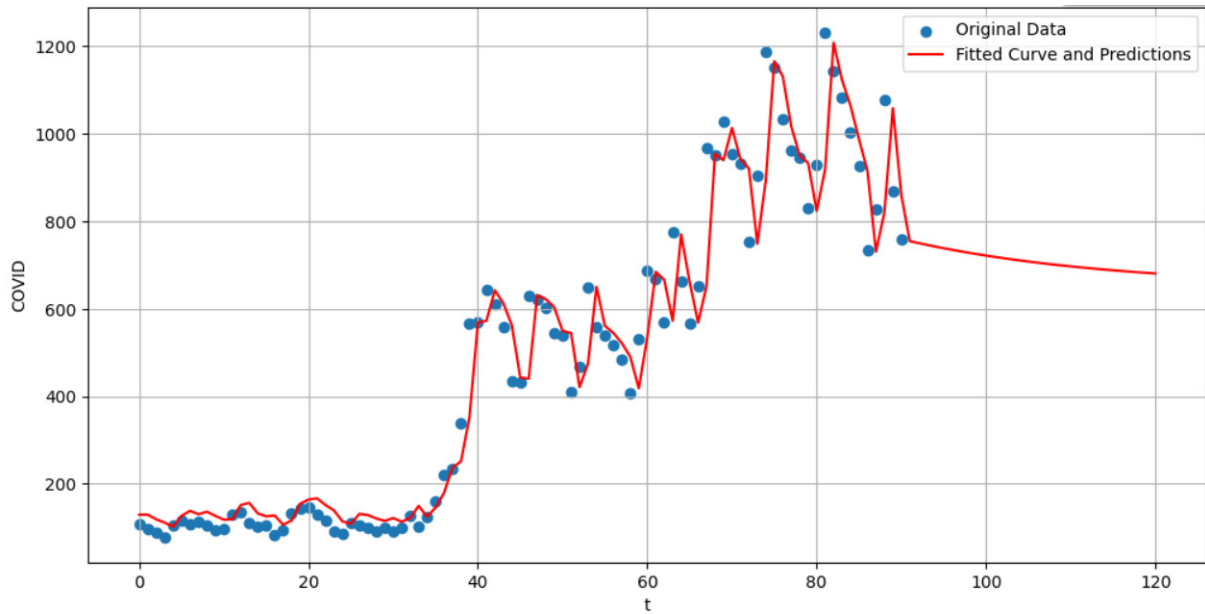


Figure 13. predictions for COVID data by using the study model for the next thirty days.

The next table, table(1) show that the predictions for COVID data by using the proposed study model for the next thirty days.

t	prediction of COVID	t	prediction of COVID	t	prediction of COVID
1-jul-22	755	11-jul-22	719	21-jul-22	695
2-jul-22	750	12-jul-22	716	22-jul-22	693
3-jul-22	746	13-jul-22	713	23-jul-22	691
4-jul-22	742	14-jul-22	710	24-jul-22	689
5-jul-22	738	15-jul-22	708	25-jul-22	688
6-jul-22	735	16-jul-22	705	26-jul-22	686
7-jul-22	731	17-jul-22	703	27-jul-22	685
8-jul-22	728	18-jul-22	701	28-jul-22	683
9-jul-22	725	19-jul-22	699	29-jul-22	682
10-jul-22	722	20-jul-22	697	30-jul-22	681

Table 1. Prediction of COVID by using study model

6. Conclusions

The study investigates the determination of stability conditions for the proposed model using the local linear approximation method, which focuses on identifying the unique non-zero point of the model and its associated stability constraints, supported by a numerical example. Structurally, the proposed model bears a strong resemblance to the exponential autoregressive model developed by the Japanese researcher Ozaki, as it integrates both a linear component and a nonlinear component, the latter characterized by a decreasing function. The model was validated with real-world data, specifically the daily counts of new COVID-19 cases in Saudi Arabia over a continuous three-month span in 2022. Parameter estimation of the first-order model was carried out in Python to ensure compliance with the stability condition at the singular point. The forecasts within the study period demonstrated high accuracy, reflected in minimal residuals (Appendix B) and robust statistical performance. A key finding of this research is that the proposed nonlinear time series model exhibits strong predictive capabilities for anticipating the future dynamics of critical phenomena, such as daily COVID-19 infections, thereby supporting the design and implementation of timely public health interventions to curb their spread and mitigate their consequences.

Appendix (A)

The COVID series data between 01-Apr-2022 and 30-Jun-2022 in the Kingdom of Saudi Arabia.

t	COVID	t	COVID	t	COVID
01-Apr-22	107	01-May-22	90	31-May-22	686
02-Apr-22	96	02-May-22	99	01-Jun-22	667
03-Apr-22	88	03-May-22	128	02-Jun-22	569
04-Apr-22	78	04-May-22	102	03-Jun-22	775
05-Apr-22	104	05-May-22	124	04-Jun-22	662
06-Apr-22	116	06-May-22	159	05-Jun-22	565
07-Apr-22	108	07-May-22	219	06-Jun-22	652
08-Apr-22	114	08-May-22	234	07-Jun-22	967
09-Apr-22	104	09-May-22	339	08-Jun-22	952
10-Apr-22	95	10-May-22	565	09-Jun-22	1029
11-Apr-22	96	11-May-22	569	10-Jun-22	955
12-Apr-22	130	12-May-22	642	11-Jun-22	932
13-Apr-22	135	13-May-22	611	12-Jun-22	753
14-Apr-22	110	14-May-22	559	13-Jun-22	905
15-Apr-22	103	15-May-22	434	14-Jun-22	1188
16-Apr-22	105	16-May-22	431	15-Jun-22	1152
17-Apr-22	83	17-May-22	630	16-Jun-22	1033
18-Apr-22	93	18-May-22	621	17-Jun-22	963
19-Apr-22	133	19-May-22	602	18-Jun-22	945
20-Apr-22	143	20-May-22	545	19-Jun-22	831
21-Apr-22	146	21-May-22	540	20-Jun-22	930
22-Apr-22	130	22-May-22	411	21-Jun-22	1232
23-Apr-22	117	23-May-22	467	22-Jun-22	1143
24-Apr-22	91	24-May-22	650	23-Jun-22	1082
25-Apr-22	85	25-May-22	557	24-Jun-22	1002
26-Apr-22	109	26-May-22	540	25-Jun-22	927
27-Apr-22	106	27-May-22	516	26-Jun-22	734

t	COVID	t	COVID	t	COVID
28-Apr-22	98	28-May-22	483	27-Jun-22	827
29-Apr-22	92	29-May-22	408	28-Jun-22	1076
30-Apr-22	99	30-May-22	530	29-Jun-22	869
				30-Jun-22	759

Appendix (B)

Log(COVID) values, forecasting values, and residuals using the nonlinear study model.

t	Log(y_t)	Forecast(Log(y_t))	Residual	t	Log(y_t)	Forecast(Log(y_t))	Residual
01-Apr-22	4.672829	4.716431	-0.0436	17-May-22	6.44572	6.081646	0.364074
02-Apr-22	4.564348	4.716431	-0.15208	18-May-22	6.431331	6.453612	-0.02228
03-Apr-22	4.477337	4.610135	-0.1328	19-May-22	6.400257	6.439513	-0.03926
04-Apr-22	4.356709	4.524876	-0.16817	20-May-22	6.300786	6.409065	-0.10828
05-Apr-22	4.644391	4.406678	0.237713	21-May-22	6.291569	6.311597	-0.02003
06-Apr-22	4.753590	4.688566	0.065024	22-May-22	6.018593	6.302566	-0.28397
07-Apr-22	4.682131	4.795565	-0.11343	23-May-22	6.146329	6.035088	0.111241
08-Apr-22	4.736198	4.725546	0.010653	24-May-22	6.476972	6.160252	0.316721
09-Apr-22	4.644391	4.778524	-0.13413	25-May-22	6.322565	6.484235	-0.16167
10-Apr-22	4.553877	4.688566	-0.13469	26-May-22	6.291569	6.332938	-0.04137
11-Apr-22	4.564348	4.599875	-0.03553	27-May-22	6.246107	6.302566	-0.05646
12-Apr-22	4.867534	4.610135	0.257399	28-May-22	6.180017	6.258019	-0.078
13-Apr-22	4.905275	4.907215	-0.00194	29-May-22	6.011267	6.19326	-0.18199
14-Apr-22	4.700480	4.944195	-0.24371	30-May-22	6.272877	6.02791	0.244967
15-Apr-22	4.634729	4.743525	-0.1088	31-May-22	6.530878	6.28425	0.246627
16-Apr-22	4.653960	4.679098	-0.02514	01-Jun-22	6.50279	6.537054	-0.03426
17-Apr-22	4.418841	4.697942	-0.2791	02-Jun-22	6.34388	6.509532	-0.16565
18-Apr-22	4.532599	4.467559	0.065041	03-Jun-22	6.652863	6.353824	0.299039
19-Apr-22	4.890349	4.579026	0.311323	04-Jun-22	6.495266	6.656583	-0.16132
20-Apr-22	4.962845	4.929570	0.033275	05-Jun-22	6.336826	6.502159	-0.16533
21-Apr-22	4.983607	5.000605	-0.017	06-Jun-22	6.480045	6.346911	0.133134
22-Apr-22	4.867534	5.020949	-0.15341	07-Jun-22	6.874198	6.487245	0.386953
23-Apr-22	4.762174	4.907215	-0.14504	08-Jun-22	6.858565	6.873460	-0.01489
24-Apr-22	4.510860	4.803976	-0.29312	09-Jun-22	6.936343	6.858141	0.078202
25-Apr-22	4.442651	4.557724	-0.11507	10-Jun-22	6.861711	6.934352	-0.07264
26-Apr-22	4.691348	4.490890	0.200458	11-Jun-22	6.837333	6.861224	-0.02389
27-Apr-22	4.663439	4.734577	-0.07114	12-Jun-22	6.624065	6.837337	-0.21327
28-Apr-22	4.584967	4.707230	-0.12226	13-Jun-22	6.807935	6.628365	0.17957
29-Apr-22	4.521789	4.630339	-0.10855	14-Jun-22	7.080027	6.808531	0.271496
30-Apr-22	4.595120	4.568433	0.026687	15-Jun-22	7.049255	7.075142	-0.02589
01-May-22	4.499810	4.640287	-0.14048	16-Jun-22	6.940222	7.044990	-0.10477
02-May-22	4.595120	4.546897	0.048223	17-Jun-22	6.870053	6.938154	-0.06810
03-May-22	4.852030	4.640287	0.211743	18-Jun-22	6.851185	6.869398	-0.01821
04-May-22	4.624973	4.892023	-0.26705	19-Jun-22	6.722630	6.850910	-0.12828
05-May-22	4.820282	4.669539	0.150743	20-Jun-22	6.835185	6.724944	0.110241
06-May-22	5.068904	4.860914	0.207991	21-Jun-22	7.116394	6.835232	0.281162
07-May-22	5.389072	5.104528	0.284543	22-Jun-22	7.041412	7.110777	-0.06937
08-May-22	5.455321	5.418247	0.037074	23-Jun-22	6.986566	7.037305	-0.05074
09-May-22	5.826000	5.483162	0.342838	24-Jun-22	6.909753	6.983564	-0.07381
10-May-22	6.336826	5.846375	0.490451	25-Jun-22	6.831954	6.908298	-0.07634
11-May-22	6.343880	6.346911	-0.00303	26-Jun-22	6.598509	6.832066	-0.23356
12-May-22	6.464588	6.353824	0.110765	27-Jun-22	6.717805	6.603323	0.114481
13-May-22	6.415097	6.472100	-0.05700	28-Jun-22	6.981006	6.720216	0.26079

t	Log(y_t)	Forecast(Log(y_t))	Residual	t	Log(y_t)	Forecast(Log(y_t))	Residual
14-May-22	6.326149	6.423606	-0.09746	29-Jun-22	6.767343	6.978116	-0.21077
15-May-22	6.073045	6.336450	-0.26341	30-Jun-22	6.632002	6.768757	-0.13675
16-May-22	6.066108	6.088443	-0.02233				

Appendix (C)

COVID values, forecasting COVID values, and residuals by using the nonlinear study model.

t	COVID	Forecast(COVID)	Residual	t	COVID	Forecast(COVID)	Residual
01-Apr-22	107	129	-22	17-May-22	630	440	190
02-Apr-22	96	129	-33	18-May-22	621	631	-10
03-Apr-22	88	119	-31	19-May-22	602	622	-20
04-Apr-22	78	111	-33	20-May-22	545	604	-59
05-Apr-22	104	101	3	21-May-22	540	549	-9
06-Apr-22	116	126	-10	22-May-22	411	544	-133
07-Apr-22	108	138	-30	23-May-22	467	421	46
08-Apr-22	114	130	-16	24-May-22	650	474	176
09-Apr-22	104	136	-32	25-May-22	557	650	-93
10-Apr-22	95	126	-31	26-May-22	540	561	-21
11-Apr-22	96	118	-22	27-May-22	516	544	-28
12-Apr-22	130	119	11	28-May-22	483	521	-38
13-Apr-22	135	151	-16	29-May-22	408	490	-82
14-Apr-22	110	156	-46	30-May-22	530	418	112
15-Apr-22	103	132	-29	31-May-22	686	535	151
16-Apr-22	105	125	-20	01-Jun-22	667	684	-17
17-Apr-22	83	127	-44	02-Jun-22	569	666	-97
18-Apr-22	93	106	-13	03-Jun-22	775	572	203
19-Apr-22	133	116	17	04-Jun-22	662	770	-108
20-Apr-22	143	154	-11	05-Jun-22	565	661	-96
21-Apr-22	146	164	-18	06-Jun-22	652	568	84
22-Apr-22	130	167	-37	07-Jun-22	967	652	315
23-Apr-22	117	151	-34	08-Jun-22	952	954	-2
24-Apr-22	91	139	-48	09-Jun-22	1029	940	89
25-Apr-22	85	114	-29	10-Jun-22	955	1013	-58
26-Apr-22	109	108	1	11-Jun-22	932	942	-10
27-Apr-22	106	131	-25	12-Jun-22	753	920	-167
28-Apr-22	98	128	-30	13-Jun-22	905	749	156
29-Apr-22	92	121	-29	14-Jun-22	1188	895	293
30-Apr-22	99	115	-16	15-Jun-22	1152	1166	-14
01-May-22	90	121	-31	16-Jun-22	1033	1131	-98
02-May-22	99	113	-14	17-Jun-22	963	1017	-54
03-May-22	128	121	7	18-Jun-22	945	950	-5
04-May-22	102	149	-47	19-Jun-22	831	933	-102
05-May-22	124	124	0	20-Jun-22	930	824	106
06-May-22	159	145	14	21-Jun-22	1232	919	313
07-May-22	219	179	40	22-Jun-22	1143	1208	-65
08-May-22	234	237	-3	23-Jun-22	1082	1123	-41
09-May-22	339	251	88	24-Jun-22	1002	1064	-62
10-May-22	565	352	213	25-Jun-22	927	988	-61
11-May-22	569	568	1	26-Jun-22	734	916	-182
12-May-22	642	572	70	27-Jun-22	827	731	96
13-May-22	611	642	-31	28-Jun-22	1076	820	256
14-May-22	559	613	-54	29-Jun-22	869	1059	-190
15-May-22	434	563	-129	30-Jun-22	759	860	-101

t	COVID	Forecast(COVID)	Residual	t	COVID	Forecast(COVID)	Residual
16-May-22	431	443	-12				

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