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# Streamlined Randomized Response Model Designed to Estimate Extremely Confidential Attributes

Ahmad M. Aboalkhair [1,6], El-Emam El-Hosseiny [2,*], Mohammad A. Zayed [3,*], Tamer Elbayoumi [1,4], Mohamed Ibrahim [5,6], A. M. Elshehawey [5]

[1] *Department of Applied Statistics and Insurance, Faculty of Commerce, Mansoura University, Egypt*
[2] *Department of Insurance and Risk Management, College of Business, Imam Mohammad Ibn Saud Islamic University (IMSIU), Saudi Arabia*
[3] *Department of Mathematics and Statistics, College of Science, Imam Mohammad Ibn Saud Islamic University (IMSIU), Saudi Arabia*
[4] *Mathematics and Statistics Department, North Carolina A&T State University, 1601 East Market Street, Greensboro, NC 27411, USA*
[5] *Department of Applied, Mathematical & Actuarial Statistics, Faculty of Commerce, Damietta University, Egypt*
[6] *Department of Quantitative Methods, College of Business, King Faisal University, Saudi Arabia*

**Abstract** When addressing highly sensitive topics, respondents may provide incomplete or untruthful disclosures, compromising data accuracy. To mitigate this issue, this study introduces an innovative and efficient randomized response framework designed to enhance the estimation of highly sensitive attributes. The proposed model refines Aboalkhair's (2025) framework, which has been established as an effective alternative to Warner's and Mangat's models. This study evaluates the conditions under which the new model achieves greater efficiency than existing approaches. Through theoretical analysis and numerical simulations, accounting for partial truthful reporting, the results demonstrate the model's superior efficiency. Additionally, the paper quantifies the privacy protection level afforded by the new approach.

**Keywords** Randomized response technique, response error, privacy protection, confidential attributes, incomplete truthfulness

**AMS 2010 subject classifications** 62D05

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## 1. Introduction

In survey research, participants often exhibit nonresponse behavior or deliberate misreporting when addressing sensitive questions due to privacy concerns, leading to evasive response bias. To resolve this challenge, Warner [27] pioneered the randomized response technique (RRT), a method designed to safeguard participant confidentiality. This approach enables researchers to gather reliable data on sensitive topics while significantly reducing response bias. In this approach, respondents answer probabilistically selected questions without disclosing which one they received, ensuring their true status remains hidden. Since respondents are presumed to answer truthfully under this framework, the collected data maintains sufficient reliability for robust statistical inference.

Although Warner's method enables privacy-preserving collection of sensitive data, the randomization process introduces higher variance in estimating the population proportion with the sensitive trait. Building on Warner's

*Correspondence to: El-Emam El-Hosseiny (Email: eaalhabashy@imamu.edu.sa). Department of Insurance and Risk Management, College of Business, Imam Mohammad Ibn Saud Islamic University (IMSIU), Saudi Arabia.,
Mohammad A. Zayed (Email: maazayed@imamu.edu.sa). Department of Mathematics and Statistics, College of Science, Imam Mohammad Ibn Saud Islamic University (IMSIU), Saudi Arabia.

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pioneering approach, subsequent studies have developed enhanced randomized response technique (RRT) to mitigate this drawback. These developments have primarily focused on two key objectives: reducing estimation variance and enhancing model efficiency. Some approaches have refined Warner's initial model by optimal parameter selection for variance reduction, while others have introduced alternative estimation techniques [5, 22, 13, 14, 7, 17, 10, 25, 16, 15, 12, 29]. Most recent work has focused on making the technique more effective through innovative design modifications [11, 21, 20, 9, 26, 8, 24, 23, 28, 1, 2, 3].

Aboalkhair et al. [4], via design modification approach, proposed an efficient and user-friendly randomized response model to reliably estimate concealment behavior. Their initial study operated under the assumption of full respondent honesty. However, when addressing highly sensitive topics, complete truthful reporting cannot always be guaranteed. Recognizing this limitation, we developed an enhanced version of Aboalkhair's model specifically adapted to account for potential respondent dishonesty. This modified approach significantly improves the model's practical utility for accurately measuring highly sensitive attributes while maintaining respondent privacy.

## 2. Pioneering models

### 2.1. Warner's model

Warner [27] introduced the pioneering randomized response technique (RRT) to estimate the population proportion ($\pi$) possessing a sensitive attribute ($A$). The estimator in Warner's model is mathematically expressed as:

$$\widehat{\pi}_w = [\hat{\alpha} - q_1][1 - 2q_1]^{-1}, \qquad q_1 \neq 0.5 \tag{1}$$

where $\widehat{\alpha}$ represents the sample proportion of "Yes" responses.
with estimation variance:

$$V(\widehat{\pi}_w) = \pi(1-\pi)/n + q_1[1-q_1][1-2q_1]^{-2}/n \tag{2}$$

Greenberg et al. [11] investigated incomplete truthfulness scenarios under Warner's [27] framework. Their analysis demonstrated that the modified estimator $\widehat{\pi'}_w$ exhibits bias, with its mean squared error (MSE) given by:

$$MSE(\widehat{\pi'}_w) = \frac{\pi T(1-\pi T)}{n} + \frac{p_1(1-p_1)}{n[1-2(1-p_1)]^2} + \pi^2(T-1)^2 \tag{3}$$

where T represents the truth-telling probability.

### 2.2. Mangat's model

Mangat [20] introduced a simple randomized response approach where participants are asked to respond with "yes" if they have the sensitive attribute (A). Otherwise, they are guided to employ the Warner randomization method. In his model, the calculation of $\pi$ is outlined as:

$$\hat{\pi}_M = [\hat{\alpha} - 1 + p_1][p_1]^{-1} \tag{4}$$

with estimation variance:

$$V(\hat{\pi}_M) = \pi(1-\pi)/n + (1-\pi)q_1[1-q_1]^{-1}/n \tag{5}$$

Mangat further analyzed cases of partial truthful reporting, proving that $\hat{\pi}_M$ becomes biased. The derived mean square error is:

$$MSE(\hat{\pi}_M) = \frac{\pi T(1-\pi T)}{n(1-q_1)^2} + \frac{q_1(1-\pi)[1-q_1(1-\pi)-2\pi T]}{n(1-q_1)^2} + \frac{\pi^2(T-1)^2}{(1-q_1)^2} \tag{6}$$

### 2.3. Aboalkhair's model

Aboalkhair et al. [4] proposed a practical and effective randomized response model for estimating concealment behavior with enhanced reliability. In their model, each participant is given sets of "Yes" and "No" cards alongside a two-stage randomization tool. Participants select a "Yes" card if they have the sensitive attribute; otherwise, they are guided to use the two-stage random device. In the first stage, they are presented with the option to choose a "No" card (with probability $p_2$) or proceed to the next stage (with probability $q_2$). If they progress to the next stage, they are then faced with a choice between selecting a "No" card (with probability $p_1$) or a "Yes" card (with probability $q_1$). As shown by Aboalkhair et al., the 'Yes' response probability ($\alpha$) is given by:

$$\alpha = \pi + (1-\pi)q_1q_2 \tag{7}$$

where:
$\pi$ : Population proportion possessing the sensitive attribute.
$p_{3-s}$: Probability of selecting "No" in stage $s$, $s$ = 1,2 with $p_{3-s} + q_{3-s} = 1$.

And the estimator for $\pi$ is:

$$\hat{\pi} = [\hat{\alpha} - q_1q_2][1-q_1q_2]^{-1}, \qquad q_1q_2 \neq 0.5 \tag{8}$$

where $\hat{\alpha}$ represents the sample proportion of "Yes" responses.

When all respondents answer truthfully, the estimator variance is:

$$V(\hat{\pi}) = \frac{\pi(1-\pi)}{n} + \frac{(1-\pi)q_1q_2[1-q_1q_2][1-q_1q_2]^{-2}}{n} \tag{9}$$

While Aboalkhair et al. [4] assumed perfect truthful reporting, we subsequently examine the more realistic scenario of partial truthful disclosure.

## 3. The proposed model

The proposed model follows a methodology similar to Aboalkhair's approach, but it introduces the key difference of accounting for incomplete truthful reporting. Here, let T represent the probability that a respondent possessing a sensitive attribute answers truthfully. Notably, respondents without the sensitive attribute have no motivation to provide false answers.

Under these conditions, the 'Yes' response probability ($\alpha'$) is given by:

$$\alpha' = \pi T + (1-\pi)q_1q_2 \tag{10}$$

The estimated proportion $\widehat{\pi'}$ of individuals possessing the sensitive attribute is calculated as:

$$\widehat{\pi'} = [\widehat{\alpha'} - Q][1-Q]^{-1}, \qquad Q \neq 0.5 \tag{11}$$

Where $(\widehat{\alpha'})$ represents the sample proportion of "Yes" responses, and $Q = \Pi_1^2\, q_{3-s}$.

### 3.1. Statistical Characteristics of the Suggested Estimator

The following theorems will address the statistical properties (bias and variance) of $(\widehat{\pi'})$ introduced in Eq. (11).

Theorem 1: The estimator bias for $(\widehat{\pi'})$ is:

$$B(\widehat{\pi'}) = \frac{\pi(T-1)}{(1-Q)} \tag{12}$$

Proof:

$$B(\widehat{\pi'}) = E[\widehat{\pi'} - \pi] = E(\widehat{\pi'}) - \pi \tag{13}$$

Given that $\widehat{\alpha'} \sim Bin(n,\ \alpha')$, the bias of Eq. (11) can be expressed as:

$$B(\widehat{\pi'}) = [\alpha' - \alpha][1 - Q]^{-1} \tag{14}$$

From Eqs. (10) and (7), we derive:

$$\alpha' - \alpha = \pi(T - 1) \tag{15}$$

Substituting Eq. (15) into Eq. (14) yields the result in Eq. (12) □

Theorem 2: The estimator of the variance of $\widehat{\pi'}$ is:

$$V(\widehat{\pi'}) = \frac{\pi T(1 - \pi T)}{n(1 - Q)^2} + \frac{Q(1 - \pi)[1 - Q(1 - \pi) - 2\pi T]}{n(1 - Q)^2} \tag{16}$$

Proof:
Using Eq. (11), the variance of $(\widehat{\pi'})$ is derived as:

$$V(\widehat{\pi'}) = V([\widehat{\alpha'} - Q][1 - Q]^{-1}) = V(\widehat{\alpha'})[1 - Q]^{-2} \tag{17}$$

Given that $n\widehat{\alpha'} \sim Bin(n, \alpha')$,

$$V(\widehat{\alpha'}) = \alpha'(1 - \alpha')/n \tag{18}$$

Substituting Eq. (18) into Eq. (17) yields:

$$V(\widehat{\alpha'}) = \alpha'(1 - \alpha')[1 - Q]^{-2}/n \tag{19}$$

Using Eq. (10), we expand $\alpha'(1 - \alpha')$ as:

$$\alpha'(1 - \alpha') = [\pi T + (1 - \pi)Q][(1 - \pi T) + (1 - \pi)Q]$$

$$= \pi T(1 - \pi T) + Q(1 - \pi)[1 - Q(1 - \pi) - 2\pi T] \tag{20}$$

Finally, substituting Eq. (20) into Eq. (19) produces the result shown in Eq. (16) □

Theorem 3: The estimator mean square error for $(\widehat{\pi'})$ is:

$$MSE(\widehat{\pi'}) = \frac{\pi T(1 - \pi T)}{n(1 - Q)^2} + \frac{Q(1 - \pi)[1 - Q(1 - \pi) - 2\pi T]}{n(1 - Q)^2} + \frac{\pi^2(T - 1)^2}{(1 - Q)^2} \tag{21}$$

Proof:

$$MSE(\widehat{\pi'}) = \{V(\widehat{\pi'}) + [B(\widehat{\pi'})^2]\} \tag{22}$$

Substituting the variance expression from Eq. (16) and the bias term from Eq. (12) into Eq. (22) directly yields the result shown in Eq. (21).

### 3.2. *Efficiency Comparison*

The proposed model introduces a refinement to Aboalkhair's framework [4], which has been validated as a proficient substitute for Warner's and Mangat's randomized response models. Previous efficiency analyses by Mangat [20] demonstrated that his model surpasses Warner's [27] in terms of statistical efficiency. Consequently, our focus will center on comparing the efficiency of Mangat's model [20] with that of the proposed approach.
The suggested estimator exhibits enhanced effectiveness compared to Mangat's estimator under conditions of partial truthfulness if and only if:

$$MSE(\widehat{\pi'}) < MSE(\widehat{\pi}_M)$$

Applying Eq. (21) and Eq. (6), this inequality—after algebraic simplification—reduces to:

$$q_1(1-\pi)(1+q_2)+2\pi T<1$$

This result demonstrates that the proposed strategy consistently outperforms Mangat's conventional method when feasible parameter values are selected.
Figure 1 empirically validates this finding, confirming the theoretical efficiency advantage of the proposed approach. The figure compares the efficiency of the proposed model with Mangat's model across different parameter combinations: a sample size of $n = 100$, population proportions of individuals with the sensitive attribute $\pi = 0.01, 0.05, 0.10, 0.20$, truth-telling probability $T = 0.95, 0.90, 0.70, 0.50$ and randomization probabilities $p_1, p_2 = 0.6, 0.7, 0.8, 0.9$. The positive differences confirm the superiority of the proposed approach.

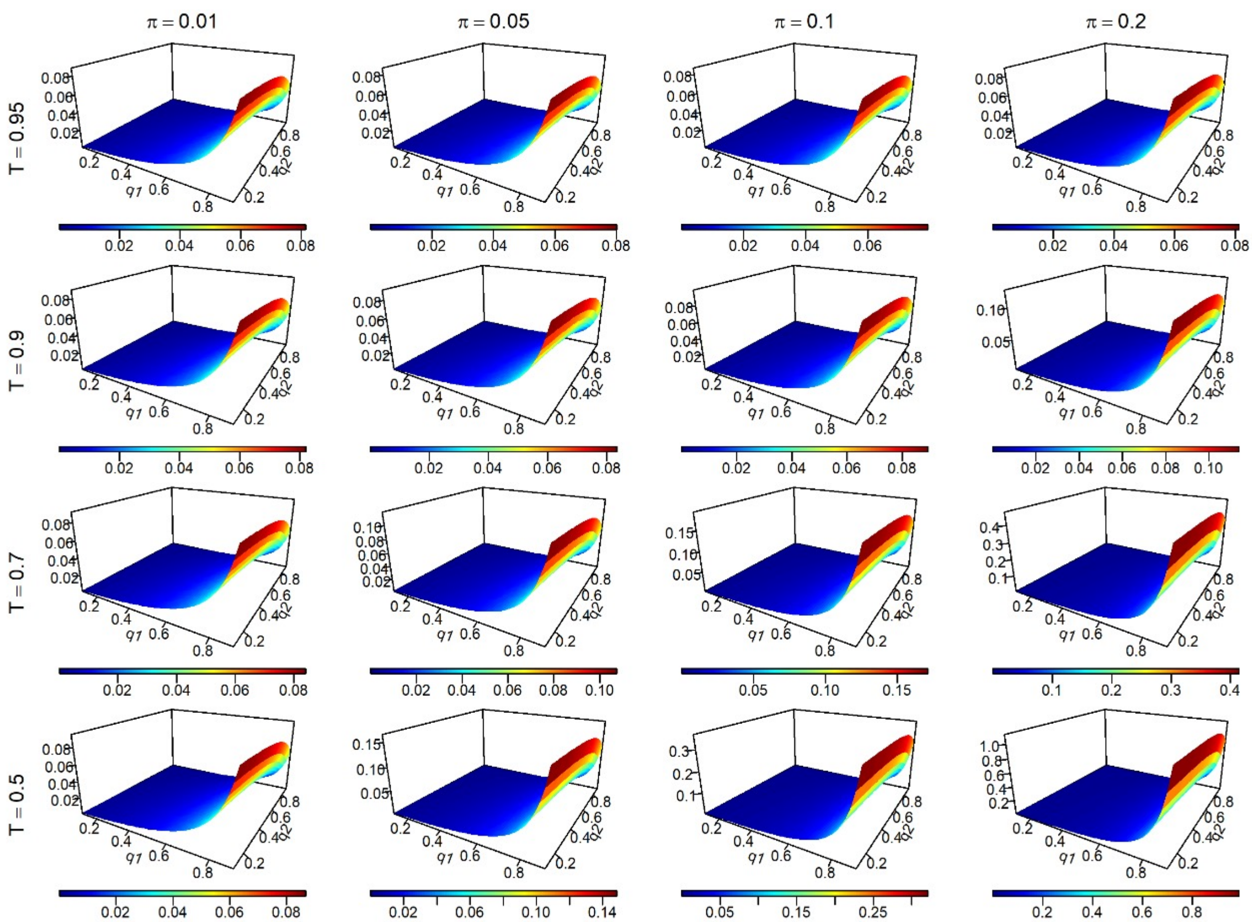


Figure 1. MSE Difference (Proposed - Mangat) across selected feasible values for $q_1, q_2, T$ and $\pi$

Figure 1 highlights the following key observations:

- For all tested values of $q_1, q_2, T$ and $\pi$, the proposed estimator is more efficient than Mangat's.
- When $q_1, q_2$ and $T$ are held constant, the efficiency gap widens as $\pi$ decreases from 0.2 to 0.01.
- For fixed $q_1, q_2$ and $\pi$, the efficiency difference increases as $T$ decreases from 0.95 to 0.50.
- With $q_2, T$ and $\pi$ fixed, the efficiency advantage grows as $q_1$ increases from 0.1 to 0.9.

- When $q_1, T$ and $\pi$ remain constant, the efficiency difference increases as $q_2$ decreases from 0.9 to 0.1. This occurs because the mean square error (MSE) of the proposed estimator decreases with lower $q_2$, whereas Mangat's MSE remains unchanged.

### 3.3. Privacy protection

Privacy protection represents a fundamental consideration in all randomized response (RR) models. Various quantitative measures for assessing privacy preservation levels in RRT have been developed in previous research [6, 18, 19, 30]. Following Zhimin and Zaizai's methodological framework [30], we derive the design probabilities as:

$$P\left(yes|A\right) = T \quad \text{and} \quad P(yes|\overline{A}) = q_1 q_2$$

$$P\left(no|A\right) = 1 - T \quad \text{and} \quad P\left(no|\overline{A}\right) = 1 - q_1 q_2$$

and

$$P\left(A|yes\right) = \frac{\pi}{\pi + (1-\pi)(Q)/T}$$

$$P\left(A|no\right) = \frac{\pi}{\pi + (1-\pi)(1-Q)/(1-T)}$$

The privacy protection metric is then defined as:

$$M_P(R) = \left|1 - \frac{1}{2}\ \{\tau(yes) + \tau(no)\}\right| \tag{23}$$

where

$$\tau(yes) = \frac{T}{Q} \quad \text{and} \quad \tau(no) = \frac{1-T}{1-Q}$$

As demonstrated by Zhimin and Zaizai [30], smaller values of $M_P(R)$in Eq. (23) correspond to stronger privacy protection for respondents. This inverse relationship indicates that minimizing the metric's value enhances participant confidentiality within the RR framework

## 4. Discussion

This study presents a methodological extension of Aboalkhair's framework [4], integrating a partial truthfulness assumption into the analytical model. While this modification specifically affects the estimator's properties and the perceived sensitivity of the studied variable, it's important to note that both models utilize the same randomization device. Consequently, they remain identical in terms of implementation, respondent burden, and empirical validation requirements. The implementation of the Aboalkhair model thoroughly explains how it estimates $\pi$ from simulated "Yes/No" responses, detailing the intermediate steps and clarifying the two-stage randomization process, along with discussing the practical implications of the parameters.

The improved efficiency of our suggested model becomes notably apparent when estimating extremely sensitive attributes that are prone to incomplete truthful reporting. This makes the model especially effective for studying extremely delicate topics— such as tax evasion, concealed wealth, socially taboo practices, health-related stigmas, criminal conduct, non-normative sexual practices, substance addiction, mental health challenges, discriminatory biases, financial misconduct, and ethical violations —areas where traditional surveys often yield incomplete or unreliable data due to participants' reluctance to share sensitive details openly, driven by privacy concerns or fear of judgment.

The randomized response technique (RRT) requires careful ethical consideration to balance the collection of sensitive data with respect for participants' rights. Researchers must ensure participants fully understand the RR method's purpose and mechanics, emphasizing their voluntary participation and right to withdraw. Transparency about data use and dissemination is equally critical, as participants should know how their responses will contribute to the study. To minimize potential distress, researchers must assess the psychological impact of sensitive questions and implement safeguards such as anonymization or access to support services. Ethical approval from institutional review boards or ethics committees is mandatory to verify compliance with welfare standards. Most importantly, participants must be assured of robust privacy protections, including guarantees that their individual responses cannot be traced or re-identified, preserving confidentiality while enabling valuable research outcomes. By addressing these concerns, researchers can uphold ethical integrity while maintaining the validity of sensitive data collection.

The probabilities $p_1$ and $p_2$ should be chosen to balance efficiency and privacy, ensuring optimal performance while reducing privacy trade-offs. This selection should aim to minimize the privacy protection metric from Eq. (23), encouraging the use of sensitive questions while lowering respondents' suspicion.

## 5. Limitations and Future Research

A key limitation of this framework is its effectiveness in scenarios involving highly sensitive attributes, particularly when respondents may not provide fully truthful answers. Traditional randomized response models, which rely on the assumption of complete honesty, become less effective in such contexts. However, when substituting Aboalkhair's model [4] in these situations, the proposed approach exhibits a limitation: its estimator has a higher Mean Squared Error (MSE) compared to Aboalkhair's method. This gap highlights an opportunity for future research to design improved randomized response techniques tailored to highly sensitive attributes, aiming to enhance accuracy by reducing MSE through two concrete strategies: (1) adaptive designs where parameters $p_1, p_2$ dynamically calibrate using preliminary response patterns, and (2) robustness testing for inter-stage dependence, particularly correlations between respondents' initial behaviors and subsequent choices.

## Author Contributions

"Conceptualization; methodology; visualization; validation; formal analysis; writing—original draft preparation; writing—review and editing; Ahmad M. Aboalkhair, El-Emam El-Hosseiny, Mohammad A. Zayed, Tamer Elbayoumi, Mohamed Ibrahim and A. M. Elshehawey; funding acquisition, El-Emam El-Hosseiny. All authors have read and agreed to the published version of the manuscript."

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability statement

The authors declare that the data supporting the findings of this study are available within the article.

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