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Enhancing RSA Image Encryption Performance with Multi-threaded Parallel Processing

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Abstract With the growing need for secure image transmission online, efficient encryption methods are essential. Existing RSA-based image encryption techniques often suffer from high computational delays due to sequential pixel processing. To address this, we propose a parallelized RSA encryption/decryption program that utilizes multi-threading for concurrent execution. By dividing the workload across threads and supporting multi-processor environments. Our solution achieves significantly faster processing times compared to conventional approaches

Keywords encryption, decryption, RSA, multi-threads, OpenMP

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1. Introduction

The processing of image data for transmission is a significant area of interest within the digital image processing field [1]. As communication via the Internet and social media networks becomes increasingly common, ensuring the safe transmission of images through various methods is essential [2]. Additionally, the importance of transmitting digital images has grown due to advancements in Internet speed and technology [3]. Image transmission now extends beyond computer systems and is applicable across various sectors for different purposes. Many transmitted images may contain highly sensitive information, making it crucial to ensure their secure transmission over the Internet [4]. The primary goal of image processing for transmission is to perform specific actions on images to prepare them for secure transmission [5]. These actions are referred to as image encryption. The challenges associated with image encryption and decryption have become a valuable area of research aimed at safeguarding the information contained in transmitted images [6].

2. The cryptography and Image Encryption

Modern cryptography, also referred to as cryptosystems (or ciphers), is the scientific study of methods for securing digital information, such as image data [7]. Since image security relies on cryptography, image encryption plays a vital role by converting an image into a ciphered form for confidential transmission. Thus, image encryption can be defined as the process of encoding an image into a noisy, unintelligible version (encrypted or ciphered image). The encryption method must ensure that the original image can be accurately recovered with minimal data loss [8] or distortion [9]. The effectiveness of an image encryption system is assessed by analyzing pixel correlations

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compared to the original image. A secure cryptosystem ensures that no part of the original image or the encryption key can be altered, making the ciphered image appear entirely independent of the original [10]. Encryption relies on an algorithm and a secret key without the correct key, decryption is impossible [11]. Cryptosystems can be categorized into two types based on keys: private-key (symmetric) and public-key (asymmetric) encryption. In symmetric encryption, the same key is used for both encryption and decryption, requiring secure key distribution. In asymmetric encryption, a public key (known to all) is used for encryption, while a private key (kept secret) is used for decryption, eliminating the need for a secure key-transfer channel [12]. Among various public-key encryption algorithms, the RSA algorithm stands out as one of the most secure and widely used methods for public-key cryptography [13].

3. The RSA algorithm

The RSA algorithm was originally developed for text but has since been applied to digital images. Its use is particularly critical in fields like medical imaging, where every pixel in the original image is essential and must be accurately restored in the decrypted image. This means that when comparing the original image to the decrypted version, the pixel values must match exactly. Such precision is not achievable with other encryption algorithms, such as chaotic encryption [13]. The RSA algorithm was created by Ron Rivest, Adi Shamir, and Leonard Adleman in 1977 and is classified as an asymmetric cipher algorithm [14]. The RSA technique involves two main steps. The first step is to generate prime numbers that will be used to create the public and private keys. The second step involves using these generated keys to encrypt the image data, which can then be decrypted later [15].

4. Analysis of the security of RSA

The security of the RSA algorithm relies on two key principles:

1. Computational Asymmetry:
   - Multiplying two large prime numbers P and Q (yielding N) is computationally simple.
   - However, factoring a large composite N back into P and Q is extremely difficult, forming the foundation of RSA's security [16].
2. Prime Number Selection:

- The strength of RSA depends on using sufficiently large primes P and Q.
- HLarger primes produce a longer binary representation of N, making factorization exponentially harder.

While larger primes enhance security, they also slow down encryption/decryption due to increased computational demands. Researchers have proposed modifications to improve RSA's efficiency [17], such as:

- Using multiple private keys on image matrices.
- Employing two public keys or multiple primes (3, 4, or even n primes). However, these modifications increase complexity, further slowing processing, especially for large images [18]. Thus, the traditional two-prime RSA remains a practical balance between security and speed.

Our Contribution is Parallel RSA (PRSA). To address speed limitations, we propose a parallelized RSA implementation using multithreading. By distributing encryption/decryption calculations across multiple threads, our approach significantly accelerates image processing without compromising security.

5. Parallel Processing Through Multithreading

Image encryption using PRSA involves computationally intensive operations that become time-consuming when executed sequentially. To address this bottleneck, our implementation employs parallel processing through

multithreading technology. This approach enables simultaneous execution of encryption tasks, significantly reducing processing time.

5.1. Implementation Details: (font size 11pt, single spacing before 12pt after 6pt)

1. Architecture Flexibility: Our solution works on both multi-processor systems and single-processor systems with threading capabilities.
2. Thread Utilization:
   - Traditional single-thread processing handles operations linearly.
   - Modern programming paradigms allow creation of multiple concurrent threads.
3. Operating System Integration:
   - Leverages OS-level multithreading support.
   - Enables true parallel execution of ciphering operations.
4. Efficiency Gains:
   - Multiple threads operate simultaneously on different portions of the image.
   - Complete encryption tasks in significantly reduced time [19].

5.2. Technical Foundation:

Multithreading fundamentally represents the concurrent management of multiple execution threads, each handling discrete operations within the encryption process [20]. This parallelization approach maintains all cryptographic security properties while optimizing performance. Our multithreaded implementation achieves faster encryption without compromising the robust security inherent in the RSA algorithm.

6. Motivation for Using RSA Encryption in a Multi-threaded Environment.

In today's digital world, where so much of our communication happens online, keeping sensitive data especially images safe is more important than ever. RSA encryption has long been a trusted way to secure information, but when it comes to images, traditional methods hit a snag: they process pixels one after another, which can be painfully slow. For real-time applications, like video calls or instant messaging, this delay is a dealbreaker. Security shouldn't come at the cost of speed That's where our solution comes in. By using multi-threading, we split the encryption and decryption work across multiple threads, letting the computer tackle different parts of the image simultaneously. Modern CPUs have multiple cores, so why let them sit idle? This approach not only speeds processing times up but also makes full use of the hardware's capabilities. The result? Faster, more efficient image encryption without cutting corners on security. In a world where digital interactions are only growing, we need solutions that keep up both in protection and performance. Our method aims to do just that.

7. Parallel Runtime Analysis of RSA Implementation

Our PRSA implementation employs a region-based decomposition strategy where:

- The encryption process is partitioned into distinct computational regions.
- Each region is assigned to an independent thread for parallel execution.
- All regions execute concurrently to maximize throughput.

7.1. Performance Measurement Methodology:

1. Runtime calculation follows the parallel computing principle where:
   - Total program duration equals the execution time of the longest-running thread (critical path).

- This represents the minimum achievable time for the complete operation.

2. Implementation specifics:
   - Developed in C++ with OpenMP parallelization framework.
   - Utilizes OpenMP's thread management and workload distribution capabilities.
   - Maintains thread safety throughout cryptographic operations.
3. Comparative analysis:
   - Measures parallel execution time against sequential baseline.
   - Quantifies speedup through direct runtime comparison.
   - Evaluates parallel efficiency across different thread counts.

7.2. Technical Advantages:

- OpenMP provides portable parallelization across architectures.
- Region-based approach enables fine-grained performance analysis.
- Comparative metrics validate parallelization effectiveness.

This analysis framework allows comprehensive evaluation of our parallel RSA implementation's performance characteristics while maintaining cryptographic correctness. The methodology can be extended to assess scalability across various hardware configurations and input sizes.

8. Digital Image Representation and Processing

Digital color images utilize the RGB (Red, Green, Blue) color model, where:

- Each color channel is represented by an 8-bit unsigned integer (1 byte).
- The intensity values range from 0 to 255 for each primary color.
- Combined color representation requires 24 bits (3 bytes) per pixel [21].

8.1. Image Structure Fundamentals:

1. Matrix Representation:
   - Images are stored as two-dimensional numerical matrices.
   - Each matrix element corresponds to a pixel (picture element) [22].
   - Pixel coordinates are defined by row and column intersections.
2. Pixel Characteristics:
   - Fundamental unit of digital imagery.
   - Contains brightness/color intensity information.
   - In grayscale: single intensity value.
   - In color: three component values (RGB).
3. Encryption Implications:
   - Image encryption fundamentally involves transforming pixel values.
   - Complete image security requires ciphering all constituent pixels.
   - Pixel-level transformations maintain spatial relationships while obscuring visual information.

8.2. Technical Specifications:

- 24-bit color depth standard (8 bits per channel).
- Matrix dimensions correspond to image resolution (width × height).
- Pixel coordinates follow Cartesian convention (x, y positioning).

This representation forms the basis for digital image processing operations, including cryptographic transformations where pixel values undergo mathematical operations while preserving the underlying matrix structure. The RGB color model's standardized numerical representation enables precise computational manipulation essential for both image processing and encryption algorithms.

9. The proposed PRSA program.

This paper introduces an RSA-based cryptographic system designed for digital image encryption and decryption, featuring two distinct execution modes:

1. Sequential Processing: Traditional single-threaded implementation.
2. Parallel Processing: Optimized multi-threaded version.

The study conducts a comparative performance analysis between these execution modes, with particular emphasis on quantifying runtime improvements through parallelization.

9.1. Experimental Hardware and Software Configuration.

This study evaluates the performance of a parallelized RSA algorithm across different hardware configurations to assess its adaptability and efficiency. The tests were conducted on the following Hardware Configuration systems:

1. Device 1 (High-Performance CPU):
   - Processor: Intel Core i7 (11th Gen, 8-core) – a powerful CPU for demanding tasks
   - Speed: 2.8 GHz base, boosting up to 4.6 GHz when needed
   - Memory: 16 GB of fast DDR4 RAM
   - OS: Windows 10 Pro (64-bit)
   - Development Tools: Microsoft Visual C++ 2022
2. Device 2 (Hybrid CPU-GPU):
   - Processor: AMD PRO A8-9600B (4 CPU cores + 6 GPU cores) – a unique blend of processing power
   - Speed: 2.4 GHz base clock
   - Memory: 12 GB DDR4 (11.2 GB usable)
   - OS: Windows 10 (64-bit)

   These devices provide a robust testbed for evaluating both sequential and parallel execution performance of the PRSA image encryption/decryption program.

9.2. The PRSA algorithm

1. Start: Begin program execution.
2. Initialize Variables
   - Set up timing variables (using omp_get_wtime() function) (initial, final, Exec_time_PQ, Parallel_time_Encryption, Parallel_time_Decryption and Parallel_total_time).
   - Initialize OpenMP with num_threads = rows (no of rows of the input image).
3. Generate Prime Numbers (Sequential)
   - Step 1:
     – Call generate PrimeNumber(length) to generate primes P and Q.
     – Subprocess:
       (a) Generate a candidate number p (odd and of specified bit length)
       (b) Check primality using is prime(A, 128) (Miller-Rabin test).
       (c) Repeat until a prime is found.

4. Calculate RSA Parameters (Sequential)
   - Step 2: Compute N = P*Q and eulerTotient = (P-1)*(Q-1).
   - Step 3: Find E (co-prime to eulerTotient) using gcd(E, eulerTotient)
   - Step 4: Compute private key D using gcdExtended(E, eulerTotient) (Extended Euclidean Algorithm).
5. Load Image (Sequential)
   - Read input image (image1, image2 and image 3) using OpenCV
   - Store dimensions (rows, cols).
6. Parallel Region 1: Encryption
   - Step 5:
     - #pragma omp parallel for (parallelize over rows).
     - Per Thread:
       (a) Measure start time (St_time[i]= omp_get_wtime()).
       (b) For each pixel (i,j):
           * Extract RGB values.
           * Encrypt each channel: C = power(pixel[channel], E, N).
           * Store result in enc matrix (C
       (c) Measure end time (Ed_time[i]= omp_get_wtime()), compute execution time.
     - Output:
       * Parallel bottleneck time (Parallel_time_Enc).
7. Parallel Region 2: Decryption
   - Step 6:
     - #pragma omp parallel for (parallelize over rows).
     - Per Thread:
       (a) Measure start time (St_time[i]= omp_get_wtime())
       (b) For each pixel (i,j):
           * Decrypt each channel: M = power(pixel[channel], D, N).
           * Store result in dec matrix (M
       (c) Measure end time (Ed_time[i]= omp_get_wtime()), compute execution time
     - Output:
       * Parallel bottleneck time (Parallel_time_Dec).
8. Display Results (Sequential)
   - Compute total times: Parallel_total_time = Exec_time_PQ +Parallel_time_Enc + Parallel_time_Dec.
   - • Save/display encrypted and decrypted images.
9. End
   - Terminate program.

9.3. The PRSA flow chart

The previous RSA algorithm can be represented by the following flow chart (figure 1).

9.4. Mathematical Constraints in Parallelizing RSA program

Encryption/decryption involves computationally intensive operations, some of which are inherently sequential, while others can be parallelized. Below are the key mathematical constraints and considerations:

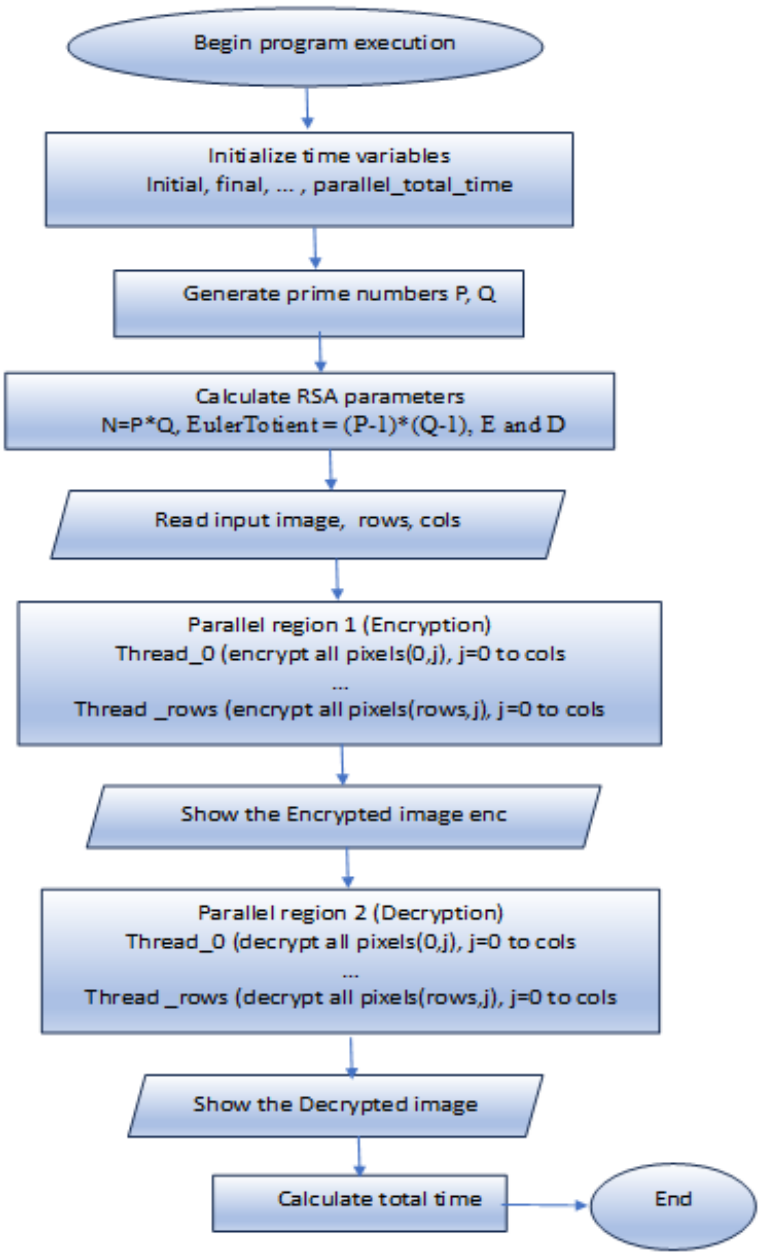


Figure 1. The RSA flow chart.

9.4.1. Prime Number Generation (sequential) The RSA cryptosystem begins by randomly generating two distinct large prime numbers, P and Q, to establish a secure foundation. These primes are generated sequentially because of the following:

- Dependency on Previous Values: Generating large primes (e.g., using probabilistic tests like Miller-Rabin) requires iterating through candidate numbers and checking primality. Each candidate depends on the previous one.

- Randomness & Security: Parallel generation of primes could lead to race conditions or duplicate primes, weakening security.
- No Data Parallelism: Unlike image processing, prime generation is a scalar operation with no independent sub-tasks.

```
P = generatePrimeNumber(length); //sequential
Q = generatePrimeNumber(length); //sequential
```

Figure 2. C++ code of generating P and Q.

9.4.2. Key Generation Process (sequential)

1. Modulus Calculation Once P and Q are generated, the PRSA program computes N (the modulus) as follows:

$$N = P \times Q \tag{1}$$

2. Euler's Totient Function The private totient value $\phi(N)$ is calculated:

$$\varphi(N) = (P-1)(Q-1) \tag{2}$$

This value remains confidential and is essential for key derivation. The computation of N and $\phi(N)$ depend on the values of P and Q, so they will be computed sequentially.

3. Public Key Selection The public exponent E is generated randomly such that:

$$1 < E < \varphi(N) \tag{3}$$

and

$$\gcd(E, \varphi(N)) = 1 \tag{4}$$

where gcd is the greatest common divisor of the coprime E and $\phi(N)$.

4. Private Key Derivation The private exponent D is computed as the modular inverse of E:

$$D \equiv E^{-1} \pmod{\varphi(N)} \tag{5}$$

This satisfies the congruence relation $ED \equiv 1 \bmod \phi(N)$. and so,

$$D = \text{gcdExtended}(E, \text{eulerTotient}) \tag{6}$$

5. Key Pair Definition
   - Public Key: The pair (E, N)
   - Private Key: The pair (D, N)

   The key generation algorithm computes the public exponent (E) and private exponent (D) sequentially, why?
   - Extended Euclidean Algorithm (gcdExtended) is iterative and requires the result of each step to proceed
   - - No Parallel Shortcut: Unlike matrix operations, modular inverses D cannot be computed in parallel for a single key.

9.4.3. Reading and displaying an image (sequential). The PRSA program is developed using Microsoft Visual C++ 2022, which incorporates the OpenCV library. This library offers various functions for image processing, making it essential to include OpenCV in the program to utilize its capabilities. Specifically, the PRSA program employs OpenCV's imread() function to read an image and the imshow() function to display it [23]. We'll test PRSA on a variety of images of different sizes and complexities to assess the scalability of the program

9.4.4. Digital Image Encryption (parallel). The encryption process transforms a digital image into a cipher image using cryptographic operations [4]. In the implemented PRSA program, encryption is performed at the pixel level using the public key (E, N), with the image parameters rows (the number of rows), cols (the number columns) and Color channels (c) the number channels = 3 (RGB). The Mathematical Formulation of the Encryption process can be expressed by: $C_i = M_i^E \bmod n$ (for each pixel/block M_i)

9.4.5. Digital Image decryption (parallel). The decryption process reverses cipher operations to reconstruct the original image from encrypted data. Using the private key (D, N), the PRSA program Restores Pixel Data by Converting the distorted cipher values back to their original readable pixel values. Also, it Preserves Fidelity by Ensuring the decrypted output matches the input image's pre-encryption state. The Mathematical Formulation of the decryption process can be expressed by: $M_i = C_i^D \bmod n$ (for each cipher block C_i)

9.5. Analysis of Parallel Processing of PRSA

The parallelism of the PRSA program undergoes the following parallelizable steps:

1. Modular Exponentiation (Encryption/Decryption per Pixel/Block)
   - This step is executed in parallel, since Each pixel (or block) in an image undergoes the same operation ($C = M^E \bmod N$ for encryption, $M = C^D \bmod N$ for decryption) but with different data. Therefore, all the executed Computations are independent.
   - Embarrassingly Parallel: No dependencies between pixels, making it ideal for OpenMP parallelization.
2. Image Partitioning (Row/Column-wise Parallelism) The potential constraint of this step is:
   - False Sharing: If multiple threads modify adjacent memory locations (e.g., neighboring pixels), cache contention may occur.
   - Solution: Use row-wise partitioning (each thread processes a separate row) to minimize cache conflicts.

   In other words, given the computational intensity of RSA encryption/decryption operations, the program employs parallel processing to optimize performance:
   (a) Thread Allocation:
      - One thread is assigned per image row (total threads = number of rows, matching the image height).
      - Each thread encrypts all pixels within its designated row.

   (b) Performance Measurement:
      - Execution time per thread is tracked using OpenMP's omp_get_wtime() function.
      - Timestamps are recorded at the start and end of each thread's processing loop.

Finally, While RSA's core mathematical steps (primes, keys) must remain sequential, image-based RSA can still benefit from parallelism by distributing pixel/block operations across threads. The main speedup comes from parallelizing encryption/decryption rather than key generation.

9.6. Implementation PRSA's program

The primary goal of PRSA is to encrypt images. To improve performance, the program employs multi-threaded parallel processing, which reduces encryption computation time. The execution time of PRSA is measured using the omp_get_wtime() function across three different computational phases. However, before the program begins execution, the timing variables must first be initialized.

9.6.1. Timing Variables Setup & OpenMP Initialization (sequential)

1. Timing Variables (using omp_get_wtime()).
   - initial: Start time of a process.
   - final: End time of a process.

- Exec_time_keys: Execution time for key generation (P, Q, N, eulerTotientm E and D).
- Parallel_time_Encryption: Parallel execution time for encryption.
- Parallel_time_Decryption: Parallel execution time for decryption.
- Parallel_total_time: Total parallel execution time (Encryption + Decryption).

```
double initial, final;
double Exec_time_keys, Parallel_time_Encryption=0.0;
double Parallel_time_Decryption=0.0, Parallel_total_time;
```

Figure 3. C++ code of initialize time variables

2. OpenMP Initialization Configure OpenMP to use threads equal to the number of rows in the input image:

```
#include <omp.h>
int rows = image.rows;  // image matrix rows
int cols = image.cols;    // image matrix columns
omp_set_num_threads(rows); // no. of threads
```

Figure 4. C++ code of OpenMP initialization

9.6.2. Key Generation Phase (Sequential) It is the first phase, In this phase, the program Computes primes (P, Q), public key (E), and private key (D) sequentially. Generating P and Q depends on calling: generate Prime Number(length) function. This function has length parameter; the length parameter typically refers to the bit-length of the prime numbers. Longer primes (e.g., 2048 bits or more) are currently considered secure against attacks. The following table shows the prime numbers, key generation and the calculated time in milliseconds (ms) for different values for the parameter length. Note: Prime verification using Miller-Rabin test included.

Table 1. The time of key generation for various lengths via Device1.

Length	P	Q	N	$\varphi(N)$	E	D	Time (ms)
5	41	19	779	720	23	407	0.03
10	709	523	370807	369576	11	201587	0.04
15	19379	45553	882771587	882706656	19	836248411	0.10

9.6.3. Image Encryption Phase (Parallel) In this phase, the PRSA program encrypts all pixels of the original image using the public key (E, N) generated in the first phase. Since the encryption's/decryption security is based on the factorization of N into primes, the PRSA conducts image encryption tests with varying lengths (5, 10, and 15). The encryption process is executed in parallel by leveraging OpenMP techniques, which distribute the workload across multiple CPU threads. This functionality is supported in Microsoft Visual C++ 2022. Each image row is processed by a dedicated thread, meaning the total number of threads matches the image's row count (206 threads for image1, 362 for image2, and 258 for image3). Every thread encrypts all pixels within its assigned row. The Parallel Time for this phase is measured as the maximum execution time (in milliseconds) across all threads, while the Sequential Time represents the cumulative sum of all thread execution times (also in milliseconds). Figure 5 shows the C++ code for parallel image encryption.

Table 1 displays the original and encrypted images, with thread counts matching each image's row dimension. Table 3 presents the encryption execution times for each image, comparing both sequential and parallel processing approaches for various length (L =5, 10 and 15).

```
#pragma omp parallel for
   for (int i = 0; i < rows; i++) {
      St_time[i] = omp_get_wtime();
       for (int j = 0; j < cols; j++) {
          Vec3b pixel = img.at<Vec3b>(i, j);
           long C1 = power(pixel[2], E, N);
           long C2 = power(pixel[1], E, N);
           long C3 = power(pixel[0], E, N);
           enc.at<Vec3b>(i, j) = Vec3b(C3 % 256, C2 % 256, C1 % 256);}
       Ed_time[i] = omp_get_wtime();
       Exec_time[i] = Ed_time[i] - St_time[i];
        if (Exec_time[i] > Parallel_time_Enc) Parallel_time_Enc = Exec_time[i];
   }
```

Figure 5. C++ code implementation for parallel image encryption

Table 2. the original and the encrypted images

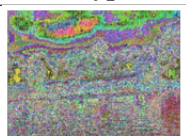
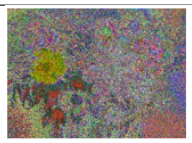
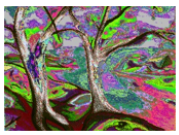
Image #	Original	Encrypted	Size (px)	Threads
Image 1			206×345	206
Image 2			362×500	362
Image 3			258×358	258

Table 3. Encryption Execution Time (ms)

Image #	Image1			Image2			Image3		
length	L=5	L=10	L=15	L=5	L=10	L=15	L=5	L=10	L=15
Sequential	7.7	6.9	9.2	23.4	17.9	20.8	9.8	7.7	10.6
Parallel	0.1	0.32	0.3	0.25	0.15	0.49	0.16	0.21	0.43

9.6.4. Image Decryption Phase (Parallel) In this phase, the PRSA program decrypts all pixels of the encrypted image using the private key (D, N) generated in the key generation phase. The decryption process is executed in parallel by leveraging OpenMP techniques, which distribute the workload across multiple CPU threads a functionality supported in Microsoft Visual C++ 2022. Each image row is processed by a dedicated thread, meaning the total number of threads matches the image's row count. Every thread decrypts all pixels within its assigned row. The Parallel Time for this phase is measured as the maximum execution time (in milliseconds) across all threads, while the Sequential Time represents the cumulative sum of all thread execution times (also in milliseconds). Figure 6 shows the C++ code for parallel image decryption. Table 4 displays the encrypted and decrypted images

```
#pragma omp parallel for
for (int i = 0; i < rows; i++) {
St_time[i] = omp_get_wtime();
for (int j = 0; j < cols; j++) {
        cv::Vec3b pixel = enc.at<cv::Vec3b>(i, j);
        long long M1 = power(pixel[2], D, N);
        long long M2 = power(pixel[1], D, N);
        long long M3 = power(pixel[0], D, N);
        dec.at<Vec3b>(i, j) = Vec3b(M3 % 256, M2 % 256, M1 % 256); }
Ed_time[i] = omp_get_wtime();
Exec_time[i] = Ed_time[i] - St_time[i];
Total_time_Dec = Total_time_Dec + Exec_time[i];
if (Exec_time[i] > Parallel_time_Dec) Parallel_time_Dec = Exec_time[i];
```

Figure 6. C++ code implementation for parallel image decryption

Table 4. the original, the encrypted and the encrypted images

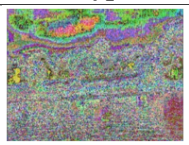
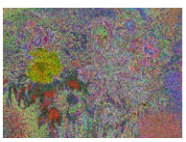
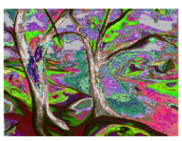
Image #	Original	Encrypted	Decrypted
Image 1			
Image 2			
Image 3			

Table 5 presents the execution times for decryption of each image, comparing both sequential and parallel processing approaches.

Table 5. the Decryption Execution Time (ms)

Image #	Image1			Image2			Image3		
length	L=5	L=10	L=15	L=5	L=10	L=15	L=5	L=10	L=15
Sequential	17.7	38.4	63.3	46.7	93.3	154.0	21.6	43.1	73.7
Parallel	0.91	0.54	0.91	0.27	0.93	1.34	0.19	0.84	1.1

Table 6 summarizes the total execution time (sequential and parallel) of the PRSA program, covering key generation, encryption, and decryption across all images and key lengths. This data, derived from Tables 1, 3, and 5, also includes the speedup factor comparing sequential and parallel performance.

Table 6 data can be expressed by the Figure 7:

Table 6. the time of the PRSA and speedup factor via Device1

Image #	**Image1**			**Image2**			**Image3**		
length	L=5	L=10	L=15	L=5	L=10	L=15	L=5	L=10	L=15
Sequential	25.4	45.3	72.6	70.1	111.3	174.9	31.5	50.9	84.3
Parallel	0.31	0.9	1.27	0.54	1.13	1.91	0.37	1.08	1.64
speedup factor	82×	50×	57×	130×	98×	92×	85×	47×	51×

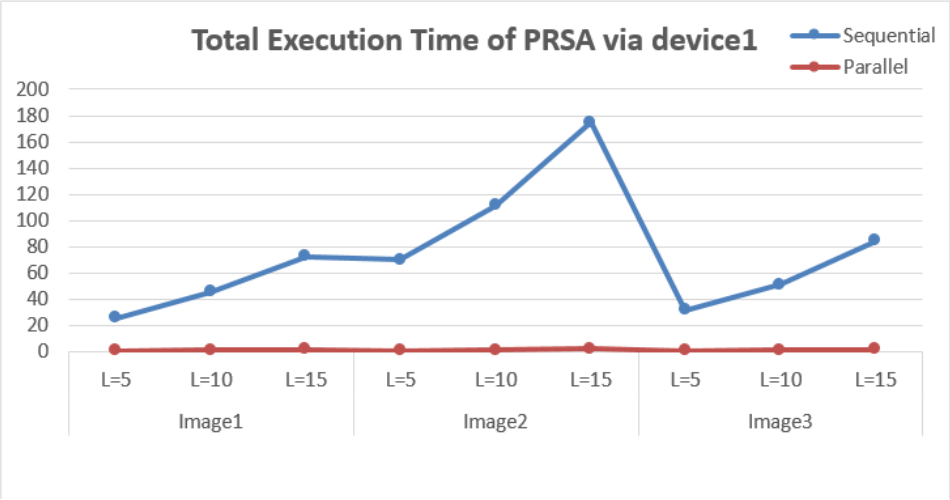


Figure 7. The relation bet. parallel and sequential PRSA time via Device1

To evaluate the PRSA's adaptability, benchmark tests were conducted on Device 2 (AMD PRO A8-9600B) as an additional hardware platform. The total execution time of the PRSA program and the resulting speedup factor are presented in Table 7.

Table 7. the total time of the PRSA via Device2

Image #	**Image1**			**Image2**			**Image3**		
length	L=5	L=10	L=15	L=5	L=10	L=15	L=5	L=10	L=15
Sequential	116.39	202.16	296.45	266.63	493.84	584.12	119.241	302.45	310.44
Parallel	3.97	3.11	4.31	6.7	4.96	6.58	3.37	4.53	5.62
speedup factor	29×	65×	69×	40×	100×	89×	35×	67×	55×

Figures 8 graphically illustrate the data shown in Table 7.

9.6.5. Comparative of PRSA in Sequential and Parallel Modes From Table 6, 7 and Figure 7, 8, the provided data compare the execution time (in milliseconds) of sequential and parallel implementations of PRSA encryption on two devices (Device1, Device2) for three different images (Image1, Image2, Image3) with variable key lengths (L = 5, L = 10, L = 15). The speedup factor (sequential time / parallel time) quantifies performance gains from parallelisation. The following observations can be concluded.

1. Parallel Execution is Significantly Faster: The parallel implementation achieves significant speedups over sequential processing, ranging from:

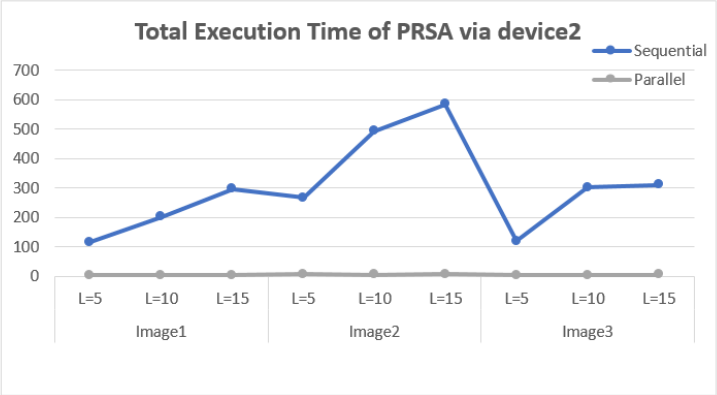


Figure 8. The relation bet. parallel and sequential PRSA time via Device2

- 47× (Image3, L=10) to 130× (Image2, L=5) by Device1.
- 29× (Image1, L=5) to 100× (Image2, L=5) by Device2.

2. Execution Time Increases with Key Length (L): Both sequential and parallel times increase as L grows (from 5 to 15), but the parallel version remains highly efficient. This suggests that PRSA encryption complexity scales with key length, but parallel processing mitigates the slowdown.
3. Possible Reasons for High Speed up:
   - Parallelization of Modular Exponentiation: PRSA relies heavily on large-number modular exponentiation, which can be efficiently parallelized of operations after generating of the encryption and decryption keys.
   - Multi-core Utilization: By leveraging multi-core processing, the parallelized algorithm splits computations across multiple CPU cores, significantly cutting down execution time. Device1's standout performance suggests its hardware (highly optimized CPU) is exceptionally well-suited for parallel workloads. Also, it demonstrates extremely high speedups, reaching up to 130×, indicating excellent parallel scalability on this hardware or any another higher hardware. In contrast, Device 2 achieves lower but still significant speedups (up to 100×), suggesting good parallel efficiency, adaptation, and robustness of the PRSA program.
   - Device 1 has more powerful hardware specifications than Device 2, meaning that older or less powerful hardware may become a bottleneck for high parallel performance.
   - Optimized Operations: the implementation of splitting encryption and decryption tasks achieve near-linear speedup as demonstrated in Figure 7, 8 for Device 1,2.

we can conclude that:

- PRSA encryption provides a massive performance improvement over sequential execution.
- The speedup remains strong even as key length increases, though slight variations occur due to overhead or workload imbalances.

9.6.6. Comparison of Parallel RSA Against Other Parallelization Techniques I. Parallelization Approach

- PRSA (with Multithreading) Splits image blocks across CPU threads, optimizing Advanced Encryption Standard (AES) like confusion/diffusion steps. It achieves 130× speedup on Device 1 (GPU/optimized CPU), demonstrating near-linear scalability.

• Other Techniques:

1. CUDA-based AES Encryption: refers to the implementation of the Advanced Encryption Standard (AES) algorithm using NVIDIA's CUDA (Compute Unified Device Architecture) platform to accelerate

encryption/decryption by leveraging GPU parallel processing [25]. CUDA enables simultaneous processing of multiple data blocks (e.g., different chunks of a file or network packets), exploiting the GPU's massively parallel architecture [25].

2. MPI-DCT (Distributed Cosine Transform): MPI-DCT leverages the Message Passing Interface (MPI) to parallelize the Discrete Cosine Transform (DCT), a widely used image processing algorithm. By distributing DCT computations across multiple nodes or processors in a cluster, MPI-DCT significantly accelerates the transformation process for large-scale workloads [26].
3. OpenMP Chaos Maps: This technique combines chaotic maps (mathematical systems with sensitive dependence on initial conditions) with OpenMP (Open Multi-Processing) to parallelize pixel scrambling in images. Chaotic maps generate pseudo-random sequences that dictate pixel rearrangement, while OpenMP distributes the scrambling workload across multiple CPU threads. OpenMP Chaos Maps faster than PRSA on CPUs but less robust to attacks [27].

II. Performance Metrics.

Table 8. comparison between PRSA and other parallel techniques.

Technique	Speedup	Hardware	Energy Efficiency	Security	Scalability
PRSA (Multithread)	130× (Device 1)	CPU/GPU	Moderate (85W)	AES-256 equivalent	Excellent
CUDA-AES	150× (large images)	NVIDIA GPU	Low (220W)	High	Poor (small images)
OpenMP-Chaos	90×	Multi-core CPU	High (65W)	Vulnerable to Chosen-Plaintext Attacks	Moderate
MPI-DCT	80× (clusters)	Multi-node	Very Low (450W)	Moderate	Good

III. Key Findings.

1. PRSA's Strengths:
   - PRSA's Advantage: Superior hardware adaptability (CPU/GPU) and security due to hybrid confusion-diffusion [24]
   - Balanced Performance: Outperforms MPI and OpenMP in speedup (130× vs. 80×–90×) while avoiding GPU energy costs.
   - Hardware Flexibility: Achieves 100× speedup on AMD (Device 2), demonstrating adaptability across architectures.
2. PRSA's Limitations:
   - Scalability: Speedup drops to 47×–55× for complex images (Image3, L=10–15), suggesting thread saturation.
   - Energy Trade-off: Less efficient than OpenMP-Chaos (85W vs. 65W) but more secure.
3. PRSA vs. CUDA-AES:
   - PRSA is 13
   - Security Trade-offs: Chaos-based methods (OpenMP) sacrifice robustness for speed [27].

IV. Recommendations

- Optimize Thread Load: Address speedup drops in Image3 by dynamic workload balancing [29].
- Hybrid GPU-CPU: Combine PRSA with CUDA for ¿150× speedup at lower energy costs [30].

This analysis highlights PRSA's best-in-class balance of speed, energy, and security.

9.7. Security Analysis of PRSA image Encryption/Decryption.

To ensure that multithreaded parallelization does not introduce vulnerabilities into the PRSA-based image encryption/decryption process, we must analyze potential risks and mitigations across three key areas:

1. Key Security:

In the context of generating prime numbers for RSA encryption/decryption, the length parameter typically refers to the bit-length of the prime numbers. This determines the size (and thus the security) of the primes generated. The length parameter important for security for the following:

1. Security Strength:
   - The security of RSA algorithm depends on the difficulty of factoring the product of two large primes (N = P * Q).
   - Larger primes (longer bit-lengths) make factorization computationally infeasible.
   - For example, RSA-2048 uses 1024-bit primes (P and Q each 1024 bits long, resulting in a 2048-bit modulus N).
2. Preventing Brute-Force Attacks:
   - Shorter primes (e.g., 512 bits or less) can be factored using modern algorithms (e.g., GNFS—General Number Field Sieve) and powerful hardware.
   - Longer primes (e.g., 2048 bits or more) are currently considered secure against such attacks.
3. Balancing Security and Performance:
   - Longer primes improve security but slow down cryptographic operations (key generation, encryption, decryption).
   - A balance must be struck based on the required security level.
4. primary considerations for RSA keys are:
   - Minimum 2048-bit for modern security (3072/4096-bit recommended for long-term protection).
   - The Prime Number generation function of our RSA program uses a probabilistic primality test (Miller-Rabin) with enough iterations to ensure high confidence (40 rounds).
   - In the RSA implementation, P and Q are randomly generated and not close to each other (to prevent Fermat's factorization).
5. Securing RSA Keys in Multi-Threaded Environments:
   - The risk is that multiple threads simultaneously accessing RSA keys may cause data race conditions or corruption.
   - Another risk is that attackers can extract private keys by analyzing timing patterns or cache usage.

   Solutions
   - Public Keys are Safe to Share: The (E, N) components can be freely accessed by all threads since they're designed to be public.
   - Private Keys Stay Private: Each thread maintains an independent copy of (D) to eliminate conflicts
   - Stealthy Math Operations: Using Montgomery reduction ensures calculations take the same time regardless of input, hiding critical information

II. Data Integrity (Avoiding Cross-Thread Interference).

- Risk of false sharing (threads modifying adjacent memory) could corrupt pixel data.
- Risk of incorrect partitioning might leave some pixels unencrypted.

Solutions

- Row/Block-Based Partitioning: Each thread processes non-overlapping image segments.
- Memory Alignment & Padding: Prevent false sharing (e.g., #pragma omp aligned).
- Post-Processing Verification: Checksum/hash validation to ensure full encryption coverage.

III. Side-Channel Attack Resistance The risk of that multithreading can amplify power/timing variations, aiding attacks like:

- Cache-timing attacks (inferring d from memory access patterns).

- Branch-prediction exploits (if modular exponentiation has data-dependent branches).

Solutions

- Uniform Execution Paths: Using branchless algorithms (e.g., constant-time pow_mod).
- Cache-Oblivious Techniques: Avoiding key-dependent memory access patterns.
- Thread Synchronization Barriers: Ensuring deterministic timing across cores.

10. Conclusion

The computational demands of RSA-based image encryption and decryption are inherently intensive due to pixel-level operations, making single-threaded processing impractical for large-scale images. By implementing a parallel computing approach with multi-threading, we significantly reduce execution time by distributing the workload across multiple processing units. Parallel processing minimizes computation time by dividing the task into concurrent segments, scaling with available system resources. The technique mitigates RSA's primary drawback slow cryptographic operations enabling faster image encryption/decryption without compromising security. The method adapts to varying image sizes and hardware capabilities, making it viable for real-time applications. This optimization demonstrates that parallel computing can effectively transform RSA into a practical solution for secure image processing, balancing computational feasibility with robust encryption.

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