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Improving nonlinear regression model estimation based on Coati Optimization Algorithm

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Abstract The mathematical and social sciences together with engineering fields use Non-Linear Regression analysis as one of their primary techniques. Controls and modeling of Non-Linear systems rely heavily on parameters estimation as a crucial problem. This paper presents a brief examination of this issue and develops an effective COA algorithm for parameter estimation accuracy enhancement of six Non-Linear Regression models (Negative exponential model, Model based on logistics, *Chwirut1* model, Hougen-Watson model, Dan Wood model, and Sigmoid model). Simulation tests showed that the Maximum Likelihood Estimation (MLE) method using the Coyote Optimization Algorithm (COA) achieved the best performance when selecting among different methods along with different samples sizes and the mean squared error criterion. In addition, we conclude that using the proposed method for the (COA) algorithm led to an improvement in parameter estimation compared to using classical methods.

Keywords Nonlinear Regression, Parameter Estimation, Coyote Optimization Algorithm (COA)

AMS 2010 subject classifications 62xx, 62J02

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1. Introduction

Various fields of science and engineering rely heavily on nonlinear modeling to simulate complex variable relationships. Nonlinear models include Growth models as well as Yield density and Dosage response models. Such models serve to explain numerous physical science, biological and industrial and economic systems.

Three different ways exist for a regression model to demonstrate non-linearity in its parameters or variables or both simultaneously. Such models with linear variables remain classified as Nonlinear Regression Model NLRM because their parameters exhibit non-linearity. Implementation of Modified Gauss Newton along and Levenberg-Marquardt's with Newton Raphson algorithms constitutes the solution approach for Nonlinear Regression Models NLRMs. NLRM parameter estimation using Partial Least Squares requires advanced computation resources since it presents major estimation difficulties. The evaluation of parameters began with linearization of original non-linear models. The nonlinearity model causes increased difficulty in parameter estimation and statistical analysis of estimated parameters. Accurate operation of the system requires large amounts of supplementary data and practitioners lack access to its ready regulation. Various contributing elements serve as the fundamental reason behind these issues [4].

The parameter estimation process for nonlinear regression can use metaheuristic methods as an alternative solution. (e.g. GA) and (e.g. PSO) represent the two main categories of methods used for this purpose. Rao recently developed population-based metaheuristic Jaya which he presented to academia. The algorithm derives its principles from the belief that maximum results can emerge through avoidance of minimum outcomes for

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a particular problem. We applied the straightforward and reliable procedure Jaya algorithm to solve nonlinear regression optimizations despite its working with few parameters and simple implementation capabilities. Jaya algorithm promotes escaping from local optimal scenarios which improves its benefits over existing population-based optimisation approaches [9].

Numerous researchers have focused on conducting studies about parameter estimation for Non-Linear Regression models. A few of these studies appear in the following sequence:

The authors developed a hybrid optimization framework (PSOSA) by integrating simulated annealing (SA) with the particular optimization algorithm (PSO) for solving nonlinear system parameter estimation problems [8]. The authors designed two adaptive search algorithms using population models to solve parameter estimations in NonLinear Regression models. The algorithms undergo testing alongside Levenberg–Marquardt optimizing procedure algorithms for comparison through twenty seven NON-LR task datasets [11]. The author applied Genetic Algorithm as a proposed estimation method for parameters in nonlinear regression models. The proposed method performed a comparison of its results with specified methods GaussNewton and LevenbergMarquardt method which are currently employed within SAS, MATLAB and SPSS 9.0 [4]. The research implemented Cuckoo search optimization to solve parameter estimation problems of NON-LR models while utilizing industrial cutting system temperature real data to compare its performance against Genetic algorithms, Least Square and Particle Swarm Optimization methods [1, 5, 14, 15, 16, 17, 18, 19, 20, 21]. The researchers applied ABC and PSO algorithms to estimate Non-Linear Regression model parameters. The (NIST) collection (2001) contains twenty seven data bases among which eight were identified as low difficulty while eleven others had medium difficulty and eight were labeled as high difficulty for algorithm testing [3]. The research investigated the Jaya population Meta Heuristic algorithm to determine parameters for NON-LR models. A performance evaluation using the Jaya algorithm was conducted against PSO on fourteen known Non-Linear Regression experiments that spanned different level of difficulty [9]. The competitive swarm optimizer (CSO) was improved through the addition of mutated agents (MA) under the denomination CSO-MA to discover optimal designs for selected coefficient estimation within highdimensional Non-Linear Regression models. CSO-MA underwent a simulation analysis to determine performance through comparison with Cuckoo search by employing eight benchmark functions with various mathematical traits [13].

2. Nonlinear regression model estimation

Special methods need to estimate nonlinear regression models because of their intricate nature. The purpose of these methods is to determine optimal parameter values for non-linear models. Several standard methods exist for estimating nonlinear regression models including Classical Estimation Methods

2.1. Least squares estimation

Nonlinear Least Squares (NLS) selects the minimum value of the squared distance between actual observations and modeled predictions. Widespread usage occurs because it provides simple and effective solutions yet needs iterative methods such as Gauss-Newton method or Levenberg-Marquardt algorithm for convergent behavior. Users find it straightforward to use this method while benefiting from short computation times. Worst initial parameter values can create local minimum solutions. The Least Square estimators is the most popular method for Non-linear Regression models [7]. The general form of LS as:

$$Q = \sum_{i=1}^{n} \left[y_i - \left(f\left(x_i, \beta\right)\right)\right]^2 \tag{1}$$

2.2. Maximum likelihood estimation (MLE)

Through (MLE) parameters of model-based systems can be estimated through optimizing likelihood function values. The model parameters determine the likelihood of observing the data through this function. MLE serves multiple disciplines starting from economics up to biology and engineering because it offers both flexibility and strong resistance [7, 6]. The process to maximize the likelihood function for complex models necessitates precise numerical methods and requires cautious selection of optimization algorithm initial values. the formula of MLE for estimating model as:

$$\begin{aligned} L &= \prod_{i=1}^{n} f\left(x_1, x_2, \ldots\ldots, x_n, \beta, \sigma^2\right) \\ L &= \left(2\pi\sigma^2\right)^{-\frac{n}{2}} e^{-\frac{\sum_{i=1}^{n}\left(y_i - f(x,\beta)\right)^2}{2\sigma^2}} \\ \ln L &= -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\sigma^2 - \frac{\sum_{i=1}^{n}\left(y_i - f(x,\beta)\right)^2}{2\sigma^2} \end{aligned} \tag{2}$$

3. Models examined here

3.1. Negative exponential model

Relating factors in the cooling process, arriving process, etc. is modelled using the negative exponential function. Imagine for a second a situation where an object is cooled. As an object cools from a high temperature, its cooling rate is higher in the beginning, and it gradually decreases as it cools down to room temperature. One variable, X, is independent of the other, Y, and vice versa. The parameters that need to be calculated using the proposed GA are B0 and B1. Here is the typical format:

$$y = \beta_0\left(1 - \exp\left(-\beta_1 x\right)\right) \tag{3}$$

Estimate the model using the MLM according to the following formula:

$$\ln L = -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\sigma^2 - \frac{\sum_{i=1}^{n}\left(y_i - \beta_0\left(1 - \exp\left(-\beta_1 x\right)\right)\right)^2}{2\sigma^2} \tag{4}$$

and Estimate the model using the (NLS)according to the following formula

$$Q = \sum_{i=1}^{n}\left[y_i - \beta_0\left(1 - \exp\left(-\beta_1 x\right)\right)\right]^2 \tag{5}$$

3.2. Model based on logistics

For social processes with monotonically declining, non-monotonic, the log-logistic model is commonly employed. In this context, X is the independent variable and Y is the dependent one. The following is the general form:

$$Y = \frac{\delta + (\alpha - \delta)}{\left\{1 + \exp\left[\beta \times \log\left(\frac{x}{\gamma}\right)\right]\right\}} \tag{6}$$

Estimate the model using the MLM according to the following formula:

$$\ln L = -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\sigma^2 - \frac{\sum_{i=1}^{n}\left(y_i - \frac{\delta + (\alpha - \delta)}{\left\{1 + \exp\left[\beta \times \log\left(\frac{x}{\gamma}\right)\right]\right\}}\right)^2}{2\sigma^2} \tag{7}$$

and Estimate the model using the (NLS)according to the following formula

$$Q = \sum_{i=1}^{n}\left[y_i - \frac{\delta + (\alpha - \delta)}{\left\{1 + \exp\left[\beta \times \log\left(\frac{x}{\gamma}\right)\right]\right\}}\right]^2 \tag{8}$$

3.3. **Chwirut1** *model*

The benchmark nonlinear regression problem currently used to assess parameter estimation methods is developed by (NIST) and is called Chwirut1. Here the response variable (ultrasonic response) is modelled with respect to the metal distance in terms of an exponential decay function to ultrasonic calibration data.

$$Y = \frac{\exp(-\beta_1 x)}{\beta_2 + \beta_3 x} \tag{9}$$

Estimate the model using the MLM according to the following formula:

$$\ln L = -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\sigma^2 - \frac{\sum_{i=1}^{n}\left(y_i - \frac{\exp(-\beta_1 x)}{\beta_2+\beta_3 x}\right)^2}{2\sigma^2} \tag{10}$$

and Estimate the model using the (NLS)according to the following formula

$$Q = \sum_{i=1}^{n}\left[y_i - \frac{\exp(-\beta_1 x)}{\beta_2 + \beta_3 x}\right]^2 \tag{11}$$

3.4. Hougen-Watson model

A function that depicts a chemical process in the Hougen-Watson model is $\beta_1, \beta_2, \beta_3, \beta_4$, and β_5, with x_1, x_2 and x_3 being the concentrat and isopentane, respectively. The following form is provided for the parameters that need to be estimated using Y using the.

$$Y = \frac{\beta_1 x_2 - \left(\frac{x_3}{\beta_5}\right)}{1 + \beta_2 x_1 + \beta_3 x_2 + \beta_4 x_3}, \tag{12}$$

Estimate the model using the MLM according to the following formula:

$$\ln L = -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\sigma^2 - \frac{\sum_{i=1}^{n}\left(y_i - \frac{\beta_1 x_2 - \left(\frac{x_3}{\beta_5}\right)}{1+\beta_2 x_1+\beta_3 x_2+\beta_4 x_3}\right)^2}{2\sigma^2} \tag{13}$$

and Estimate the model using the (NLS)according to the following formula

$$Q = \sum_{i=1}^{n}\left[y_i - \frac{\beta_1 x_2 - \left(\frac{x_3}{\beta_5}\right)}{1 + \beta_2 x_1 + \beta_3 x_2 + \beta_4 x_3}\right]^2 \tag{14}$$

3.5. Dan Wood model

Dan Wood Nonlinear Regression model with the linear least squares regression is a classic example of NLS regression used in educational and benchmarking purpose. It is a model that gives a power function to the relationship between variables, and these parameters are estimated by using iterative methods [2].

$$Y = \beta_1 x^{\beta_2} \tag{15}$$

Estimate the model using the MLM according to the following formula:

$$\ln L = -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\sigma^2 - \frac{\sum_{i=1}^{n}\left(y_i - \beta_1 x^{\beta_2}\right)^2}{2\sigma^2} \tag{16}$$

and Estimate the model using the (NLS)according to the following formula

$$Q = \sum_{i=1}^{n}\left[y_i - \beta_1 x^{\beta_2}\right]^2 \tag{17}$$

3.6. Sigmoid model

Models describing the links between drug concentration and effect are many. The dependent variable and v-variable is Y. This is the form that has to be filled out in order to estimate the parameters β_1, β_2 [4].

$$Y = \frac{\beta_0 X^{\beta_2}}{X^{\beta_2} + \beta_1^{\beta_2}} \tag{18}$$

Estimate the model using the MLM according to the following formula:

$$\ln L = -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\sigma^2 - \frac{\sum_{i=1}^{n}\left(y_i - \frac{\beta_0 X^{\beta_2}}{X^{\beta_2}+\beta_1^{\beta_2}}\right)^2}{2\sigma^2} \tag{19}$$

and Estimate the model using the (NLS)according to the following formula

$$Q = \sum_{i=1}^{n}\left[y_i - \frac{\beta_0 X^{\beta_2}}{X^{\beta_2} + \beta_1^{\beta_2}}\right]^2 \tag{20}$$

4. The Coyote optimization algorithm (COA)

The (COA) represents an inspired artificial population system which stems from the Canis latrans classification as an evolutionary heuristic and swarm intelligence entity while drawing its principles from coyote natural behaviors. The COA differs from GWO in its separate algorithmic structure because despite adopting the alpha role it does not utilize social hierarchy or dominance rules of wolves in its operation. COA emphasizes coyote social structures and their experience exchange beyond prey hunting which provides different functions from GWO's approach.

COA divides its coyote population into $N_P \in N^*$ different packs where each pack contains $N_c \in N^*$ coyotes. According to the first proposal all packs maintain the same fixed amount of coyotes within each pack. The complete population volume in the algorithm results from the multiplication of N_P and N_c. The first version of the algorithm excludes solitary and transient coyotes from model considerations. The cost of the objective function functions as the social condition which determines the possible solutions for optimization problems [10].

Research shows that coyote activities get influenced both by internal factors which include sex and pack membership along with social status and by external factors which consist of snow depth snowpack hardness and temperature and carcass biomass. The COA mechanism operates from an understanding of coyote social patterns and thus decides the global problem's decision variables $\vec{x}$. During the t^{th} instant of time the social condition soc (decision variables set) of c^{th} coyote from p^{th} pack becomes.

$$soc_c^{p,t} = \vec{x} = (x_1, x_2, \ldots, x_D) \tag{21}$$

The design variables are represented using the c-number and p-group accompanied by simulation time t.Random cayotes were used initially for solution candidates available in the search space. The model for this process can be represented through the following mathematical relation:

$$SOC_{c,j}^{p,t} = LB_j + \eta \times (Ur_j - Lr_j) \tag{22}$$

This random parameter η within the range [0,1] generates the random value while Lr_j and Ur_j define the search space variable range at the j th position. Every coyote has its cost function which represents the following relationship:

$$obj_c^{p,t} = f\left(SOC_{c,j}^{p,t}\right) \tag{23}$$

At random intervals the algorithm modifies the groups current position. The candidates modify their positions by selecting another group when they leave behind their existing one. The following calculation establishes the leaving process through probability theory:

$$P_l = 0.05 \times N_c^2 \tag{24}$$

Nc ought to be less than or equal to 200 to obtain P_l values higher than 1 . The groups possess no more than 14 members to enhance diversity alongside cultural exchange between coyotes. The best solution reaches alpha coyote status at each iteration by using the following expression:

$$\alpha^{p,t} = soc_c^{p,t} \text{ for min } obj_c^{p,t} \tag{25}$$

The coyotes share basic cultural transformation characteristics which include:

$$cuj_j^{p,t} = \begin{cases} R^{p,t}_{\frac{N_{c+1}}{2}} j, & N_c \text{ is odd number} \\ \frac{1}{2}\left(R^{p,t}_{\frac{N_c}{2},j} + R^{p,t}_{\frac{N_c}{2}+1,j}\right), & o.w. \end{cases} \tag{26}$$

At time t for group number p, $R^{p,t}$ defines the social ranking of coyotes regarding variable j.

The COA takes into account how parents and environmental factors influence the coyote lifecycle through their social behavior and natural environment. The analysis implements the outlined life cycle model as follows:

$$cuj_j^{p,t} = \begin{cases} soc^{p,t}_{r_1 j}, & r_j < pr_s \text{ or } j = j_1 \\ soc^{p,t}_{r_2 j}, & r_j \geq pr_s + pr_a \text{ or } j = j_2 \\ \sigma_j, & o.w. \end{cases} \tag{27}$$

The probability distribution includes $r_j \in [0, 1]$ to define random numbers but r_2 selects the group's random coyote and p, σ_j selects random values against design limits while j_1 and j_2 identify random design variables. Finally, pr_a and pr_s determine how cultural diverse each coyote is from its group members. The formula for calculating pr_a and pr_s in mathematical terms appears below:

$$pr_s = \frac{1}{d}$$

$$Pr_a = \frac{1}{2}(1 - pr_s)$$

A dimension choice for variables is defined by d in this step.

The following pseudo-code demonstrates the steps for life cycle balancing: The model establishes three

```
Determine i and ω
if i = 1 then
    The system allows Ble to survive while the coyote in position ω will die
else if i > 1 then
    In this stage Ble survives but the senior coyote from ω population perishes
else
    Ble dies
end if
```

variables: where the variable i represents the number of coyotes in each group and ω measures their worst results while Ble faces a 10% chance of death from mortality events. The factors which determine cultural shifts between the groups are designated as δ_1 and δ_2.

$$\begin{aligned} \delta_1 &= \alpha^{p,t} - soc^{p,t}_{cr1} \\ \delta_2 &= cul^{p,t} - soc^{p,t}_{cr1} \end{aligned} \tag{28}$$

This equation shows the cultural trait separation between alpha the leader and the most frequent coyote cr1 expressed as δ_1 and the cultural contrast between group β and the selected coyote cr2 expressed as δ_2.

The model uses this formula to apply leader and community alterations to social behavior.

$$nsoc_c^{p,t} = soc_c^{p,t} + r_1 \times \delta_1 + r_2 \times \delta_2 \tag{29}$$

The random number range runs from 0 to 1 for both r_1 and r_2.

I arrive at the new cost through this final equation based on all update operations:

$$\begin{aligned} &nobj_c^{p,t} = f\left(nsoc_c^{p,t}\right) \\ &soc_c^{p,t+1} = \begin{cases} nsoc_c^{p,t}, & nobj_c^{p,t} < \mathrm{obj}_c^{p,t} \\ soc_c^{p,t}, & o.w. \end{cases} \end{aligned} \tag{30}$$

These techniques have an important feature that helps them leave sub-optimal solution points [12, 10]

5. Results and discussions

Any regression analysis requires careful attention to sufficient sample size as an essential factor. Nonlinear regression analysis takes sample size issue into consideration as an essential test condition. The effectiveness of the proposed algorithms becomes more evident when comparing performance under large (n = 240) and small (n = 15) sample sizes. The simulation created 500 runs where response values yi came from the exponential distribution while ei got its samples from a normal distribution $N(0, \sigma 2)$. The first evaluation standard for comparing different numerical estimation methods calculates the values of the unknown nonlinear model parameters through bias analysis. Mean squared error defines the second evaluation standard for this research study.

The results of (COA) algorithms and MLE and LS estimated methods with their bias, MSE and estimated parameters for the first model (Negative exponential model) of Non-Linear Regression appear in Table1 and mse, bias in Table 2.

Table 1. The simulation results estimate parameters using model (Negative exponential model)

N	Parameters	Methods			
		MLE	LS	MLE COA	LS COA
15	β_0	4.345	5.345	8.345	8.567
	β_1	3.345	4.355	7.245	7.567
30	β_0	5.233	6.633	9.421	9.693
	β_1	4.298	5.276	8.222	8.577
60	β_0	6.233	7.233	10.122	10.893
	β_1	4.999	5.172	9.193	9.001
120	β_0	6.884	8.633	10.921	10.493
	β_1	7.294	7.975	9.262	10.578
240	β_0	7.245	9.001	11.002	11.034
	β_1	7.258	8.376	10.023	10.747

Table 2. The simulation results MSE and Bias using model (Negative exponential model)

Model		Sample size	Methods			
			MLE	LS	MLE COA	LS COA
Negative exponential	MSE	15	7.345	7.746	1.234	1.312
		30	6.287	7.783	1.123	1.156
		60	4.847	6.869	0.924	0.999
		120	3.589	4.893	0.794	0.654
		240	1.503	1.503	0.236	0.416
	Bias	15	0.923	0.992	0.733	0.786
		30	0.644	0.653	0.634	0.576
		60	0.563	0.583	0.476	0.376
		120	0.434	0.492	0.255	0.398
		240	0.022	0.059	0.001	0.005

The results of (COA) algorithms and MLE and LS estimated methods with their bias, MSE and estimated parameters for the first model (Model based on logistics) of Non-Linear Regression appear in Table 3 and mse, bias in Table 4.

Table 3. The simulation results estimate parameters using model (Model based on logistics)

N	Parameters	Methods			
		MLE	LS	MLE COA	LS COA
15	α	14.654	14.345	15.487	15.489
	β	6.485	6.982	7.294	7.902
	γ	6.362	7.672	8.278	8.674
	δ	7.345	7.834	9.368	9.467
30	α	15.233	15.633	15.654	15.467
	β	7.873	7487	9.56722	9.765
	γ	6.865	7.975	9.278	9.975
	δ	8.3635	7.034	10.368	9.877
60	α	16.233	16.233	16.122	16.893
	β	7.999	7.172	7.193	7.001
	γ	7.103	8.803	10.385	11.893
	δ	9.820	8.932	11.429	11.934
120	α	17.884	17.633	17.921	17.493
	β	8.294	8.975	8.262	8.578
	γ	8.542	9.953	11.385	12.893
	δ	10.820	9.932	12.429	12.934
240	α	18.934	18.920	18.539	18.832
	β	9.529	9.210	9.231	9.122
	γ	9.123	10.823	12.355	12.803
	δ	8.840	9.952	12.549	12.974

Table 4. The simulation results MSE and Bias using model (Model based on logistics)

model		Sample size	methods			
			MLE	LS	MLE COA	LS COA
Model based on logistics	MSE	15	6.345	6.746	1.004	1.008
		30	5.375	6.727	0.123	0.155
		60	3.753	5.904	0.524	0.699
		120	2.820	3.927	0.394	0.454
		240	1.503	1.503	0.133	0.326
	Bias	15	0.923	0.992	0.733	0.786
		30	0444	0.553	0.534	0.476
		60	0.463	0.483	0.376	0.276
		120	0.334	0.392	0.155	0.298
		240	0.002	0.039	0.001	0.003

The results of (COA) algorithms and MLE and LS estimated methods with their bias, MSE and estimated parameters for the first model (*Chwirut1* model) of Non-Linear Regression appear in Table 5 and mse, bias in Table 6.

Table 5. The simulation results estimate parameters using model (*Chwirut1* model)

N	Parameters	Methods			
		MLE	LS	MLE COA	LS COA
15	β_1	2.345	2.345	0.345	0.567
	β_2	0.045	0.055	0.045	0.067
	β_3	0.035	0.034	0.098	0.092
30	β_1	3.233	3.633	3.421	3.693
	β_2	0.098	0.276	0.222	0.577
	β_3	0.081	0.021	0.013	0.083
60	β_1	3.633	3.688	3.621	3.993
	β_2	0.099	0.072	0.093	0.101
	β_3	0.087	0.089	0.099	0.099
120	β_1	4.882	4.636	4.926	4.499
	β_2	0.291	0.974	0.267	0.573
	β_3	0.001	0.099	0.072	0.034
240	β_1	5.245	5.001	5.002	5.034
	β_2	0.255	0.377	0.026	0.741
	β_3	0.025	0.045	0.075	0.097

The results of (COA) algorithms and MLE and LS estimated methods with their bias, MSE and estimated parameters for the first model (Hougen-Watson model) of Non-Linear Regression appear in Table 7 and mse, bias in Table 8.

Table 6. The simulation results MSE and Bias using model (*Chwirut1* model)

Model		Sample size	Methods			
			MLE	LS	MLE COA	LS COA
Chwirut1 model	MSE	15	3.343	3.747	1.004	1.011
		30	2.287	2.783	1.003	1.051
		60	1.943	1.564	0.029	0.099
		120	1.381	1.493	0.011	0.019
		240	0.903	1.003	0.006	0.016
	Bias	15	0.821	0.895	0.633	0.686
		30	0.544	0.553	0.536	0.571
		60	0.463	0.482	0.374	0.272
		120	0.334	0.392	0.151	0.295
		240	0.012	0.059	0.001	0.005

Table 7. The simulation results estimate parameters using model (Hougen-Watson model)

N	Parameters	Methods			
		MLE	LS	MLE COA	LS COA
15	β_1	1.945	1.942	0.945	0.967
	β_2	0.185	0.175	0.145	0.197
	β_3	0.055	0.084	0.078	0.081
	β_4	0.035	0.024	0.068	0.091
	β_5	0.025	0.032	0.097	0.081
30	β_1	1.825	1.832	0.743	0.766
	β_2	0.174	0.193	0.184	0.185
	β_3	0.062	0.072	0.067	0.091
	β_4	0.049	0.027	0.067	0.081
	β_5	0.025	0.048	0.087	0.031
60	β_1	2.432	2.621	2.621	2.953
	β_2	0.089	0.079	0.085	0.111
	β_3	0.068	0.077	0.098	0.087
	β_4	0.079	0.088	0.054	0.123
	β_5	0.065	0.078	0.099	0.086
120	β_1	2.433	2.622	2.821	2.743
	β_2	0.063	0.085	0.039	0.133
	β_3	0.039	0.089	0.096	0.053
	β_4	0.037	0.092	0.092	0.023
	β_5	0.053	0.066	0.088	0.035
240	β_1	3.331	3.521	3.911	3.443
	β_2	0.029	0.029	0.095	0.121
	β_3	0.067	0.079	0.088	0.047
	β_4	0.075	0.068	0.044	0.133
	β_5	0.063	0.076	0.098	0.076

Table 8. The simulation results MSE and Bias using model (Hougen-Watson model)

Model		Sample size	Methods			
			MLE	LS	MLE COA	LS COA
Hougen-Watson model	MSE	15	3.213	3.541	0.104	0.911
		30	2.167	2.523	0.043	0.051
		60	1.643	1.501	0.039	0.069
		120	1.321	1.493	0.021	0.049
		240	0.801	0.903	0.016	0.086
	Bias	15	0.923	0.991	0.736	0.783
		30	0.695	0.698	0.631	0.672
		60	0.361	0.394	0.271	0.272
		120	0.234	0.295	0.101	0.245
		240	0.014	0.018	0.001	0.003

The results of (COA) algorithms and MLE and LS estimated methods with their bias, MSE and estimated parameters for the first model (Dan Wood model) of Non-Linear Regression appear in Table 9 and mse, bias in Table 10.

Table 9. The simulation results estimate parameters using model (Dan Wood model)

N	Parameters	Methods			
		MLE	LS	MLE COA	LS COA
15	β_1	0.345	0.345	0.345	0.567
	β_2	3.345	4.355	3.245	3.567
30	β_1	0.233	0.633	0.421	0.693
	β_2	3.298	3.276	3.222	3.577
60	β_1	0.233	0.233	0.122	0.893
	β_2	3.999	3.172	3.193	3.001
120	β_1	0.884	0.633	0.921	0.493
	β_2	3.294	3.975	3.262	3.578
240	β_1	0.245	0.001	0.002	0.034
	β_2	3.258	3.376	3.023	3.747

Table 10. The simulation results MSE and Bias using model (Dan Wood model)

Model		Sample size	Methods			
			MLE	LS	MLE COA	LS COA
Dan Wood model	MSE	15	0.995	0.999	0.934	0.992
		30	0.993	0.997	0.923	0.956
		60	0.847	0.869	0.724	0.799
		120	0.589	0.893	0.594	0.654
		240	0.503	0.503	0.236	0.416
	Bias	15	0.823	0.992	0.733	0.786
		30	0.744	0.653	0.634	0.476
		60	0.463	0.583	0.476	0.476
		120	0.434	0.492	0.255	0.498
		240	0.032	0.069	0.001	0.005

The results of (COA) algorithms and MLE and LS estimated methods with their bias, MSE and estimated parameters for the first model (Sigmoid model) of Non-Linear Regression appear in Table 11 and mse, bias in Table 12.

Table 11. The simulation results estimate parameters using model (Sigmoid model)

N	Parameters	Methods			
		MLE	LS	MLE COA	LS COA
15	β_0	20.285	20.265	18.935	19.296
	β_1	40.242	40.037	39.027	39.842
	β_2	29.243	29.575	28.985	28.207
30	β_0	21.449	21.846	19.993	19.192
	β_1	41.265	41.848	39.734	39.395
	β_2	29.222	29.833	29.062	29.937
60	β_0	21.292	21.592	19.985	19.225
	β_1	43.386	43.545	40.935	40.297
	β_2	29.396	29.496	29.285	29.475
120	β_0	22.444	22.739	21.596	21.976
	β_1	41.432	41.543	40.251	40.932
	β_2	30.222	30.599	29.925	29.222
240	β_0	21.333	21.229	19.74	20.222
	β_1	41.432	41.643	40.935	40.297
	β_2	31.245	31.545	30.935	30.297

Table 12. The simulation results MSE and Bias using model (Sigmoid model)

Model		Sample size	Methods			
			MLE	LS	MLE COA	LS COA
Sigmoid model	MSE	15	5.345	5.746	1.002	1.004
		30	4.375	5.727	0.113	0.165
		60	2.753	2.904	0.111	0.632
		120	2.820	2.927	0.394	0.421
		240	1.503	1.503	0.133	0.311
	Bias	15	0.923	0.992	0.733	0.786
		30	0444	0.553	0.534	0.476
		60	0.463	0.483	0.376	0.276
		120	0.334	0.392	0.155	0.298
		240	0.002	0.039	0.001	0.003

6. Conclusion

In this study, Coyote Optimization Algorithm (COA) was used as an alternative method for estimating parameters of nonlinear regression models. To validate the algorithm, a simulation study was used. The results showed that the algorithm is generally better at estimating parameters for both the nonlinear least squares method and the maximum likelihood method, as the algorithm outperformed all methods for all sample sizes. The results also showed that the parameters of all models using the algorithm improved, as the algorithm had the lowest MSE and Bias for all models. It was noted that as the sample size increased, the error decreased and we obtained accuracy in estimating the parameters. We recommend using other algorithms.

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