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Optimizing cell load regulation capability in dynamic cell manufacturing systems

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Abstract Variation in production cell load arises from machine loads exceeding their capacity and the constraints of cellular capacity. This issue has become increasingly critical in scheduling cellular manufacturing systems. In this paper, we propose a novel approach for scheduling in dynamic cellular manufacturing systems. The objective is to minimize cell load variations and associated costs while achieving a balance between internal manufacturing and subcontracting. To address this, we developed a mixed-integer linear programming (MILP) mathematical model, which was solved using LINGO 19.0 software. The model focuses on reducing cell load variation, minimizing associated costs, and optimizing the balance between internal production and subcontracting. Extensive computational experiments use medium-scale problem instances with randomly generated demand scenarios. The results demonstrate the effectiveness of the proposed model in generating optimal solutions, significantly reducing cell load variation and related costs. Furthermore, computational efficiency is notable, with solutions obtained in very low processing times. This underscores the model's practical applicability and robustness in addressing real-world scheduling challenges in cellular manufacturing systems.

Keywords Optimization, cell load variation, cell planning, material transfer, mathematical programming, Cellular manufacturing;

AMS 2010 subject classifications 90B08

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1. Introduction

Many manufacturing companies face numerous production problems, such as planning and scheduling, which weaken their productivity. Plant design, planning, and replanning are becoming increasingly important in the dynamic cellular manufacturing system. *Cellular manufacturing systems* CMS) are increasingly being demanded as an efficient way to use *group technology* (TG). These companies seek to maximize their productivity to remain competitive in an increasingly dynamic market in the age of new technology. Group technology provides the opportunity to design cellular manufacturing systems that bring flexibility, improved production efficiency, superior quality, fast delivery, and mass production.

One of the primary goals of *cellular manufacturing systems* (CMS) is to minimize intercellular or intracellular material transfer movements. Although the switch from shop floor layout to CMS can reduce material transfer movements [1], there are many problems in planning or ordering in a system when the optimal parts routes are not yet identified. In the study of CMS, two *main work-in-progress* (WIP) movements in the system are unavoidable: intracellular and intercellular. These different work-in-progress movements are intracellular, the movement of parts

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between machines in the same cell, and intercellular, which is the movement of parts between cells. What procedure minimizes movements to avoid machine overloads and queues in the other cells? To answer this question, a new approach has been developed to limit the variation in cell load in dynamic cell production systems.

Hence, a *linear integer programming model* (L-MIP) is developed to reduce the variation of the cell load. In formulating the model, all system costs will be taken into account, not forgetting the costs that make product design possible, such as setup costs, machine purchase costs, operating costs, subcontracting costs, and backorder costs. This study aims to reduce the costs of material transfer, both intercellularly and intracellularly. The model also aims to find trade-off values between in-house manufacturing and subcontracting.

2. Literature review

To provide effective part processing and cell load regulation, the part routing problem in *cellular manufacturing systems* (CMS) seeks the best set of machines for successive operations within cells. Multiple routing paths can be considered for all parts when parallel machines are available. Although advantageous, this flexibility creates problems with intracellular and intercellular mobility, which impacts material transfer costs and cell load balance. In this review, the main contributions and shortcomings in the literature on part routing, dynamic demand, cell reconfiguration, and cell utilization are critically examined.

2.1. Parts routing

Because of its significant influence on cost management and production efficiency, the part routing problem has been the main focus of researchers.[2] focused on reducing intercellular mobility and system underutilization expenses by utilizing alternative part paths. However, their method might not be as scalable when applied to complicated industrial processes in the real world. A mathematical framework for multi-routing component allocation to manage parallel machines was proposed by [3];however, their model assumes that demand is deterministic, which might not account for the uncertainty present in dynamic CMS contexts. To optimize routing, [4] discussed multiprocess planning; however, they did not investigate adaptive techniques that could react dynamically to changing cell loads. In contrast, [5] presented a model that uses absolute robustness and deviation robustness criteria to reduce both the costs of intracell and intercell movement at the same time. Despite its effectiveness, this paradigm could not be flexible enough for situations where machine capabilities or layout changes frequently occur. A genetic algorithm for creating family of components was developed by [6] to reduce intercellular movements and fluctuations in cell load. Genetic algorithms are often computationally costly despite their potential and may not work well in dynamic, real-time environments. [7] used genetic algorithms with a mixed-integer programming model to reduce manufacturing time and intercellular movement. However, using sequence-dependent setup times may add complexity, making the solution time sensitive and difficult to generalize across various CMS configurations.

2.2. Dynamics and uncertainty

Dynamic parts demand occurs when part demand in real-world CMS applications fluctuates greatly from one planning horizon. Bottlenecks and unbalanced cell loads are often the result of such demand fluctuations, fueled by market changes, product redesigns, and the launch of new products. A material transfer approach that takes into account machine uncertainty when scheduling under fluctuating product needs was put out by [8]. Although their method is informative, it may not reflect the real-time dynamics of a CMS since it presupposes static machine availability.

[9] included intercellular work-in-progress transfer and machine relocation to handle dynamic parts demands. However, their model is not flexible or scalable for systems that undergo regular layout reconfigurations. The grouping of machine parts and *group scheduling* (GS) was combined by [10]; however, the intricacies of concurrent problem solving may affect the viability of this model in hectic production settings. These studies have significantly advanced our knowledge of CMS in the face of uncertainty, but often fail to address the rapid adaptation needed in highly dynamic environments.

2.3. Cell reconfiguration

In CMS optimization, cell reconfiguration, which involves moving machines or part families to improve material flows within and between cells, remains a little-studied strategy. Although reconfiguration can improve load distribution and reduce intercellular movement costs, its implementation is often hampered by its high costs and time constraints. In their studies, [2, 4, 5] used reconfiguration techniques, but mostly in static environments. A dynamic reconfigurable D-CMS model that considers machine relocation and maximum cell size was presented by [11, 12]. Although encouraging, this strategy assumes that machines will be available and relocated at optimal times, which is impossible in situations with much variation.

A mathematical approach to CMS reconfiguration that considers the costs of replacing faulty parts and preventive maintenance was later developed by [13]. Despite being novel, this model might not be able to handle high-frequency reconfigurations. For cell construction and machine movement, [14] used a multiple traveling salesman formulation; nevertheless, their strategy might use more investigation of real-time adjustment capabilities, which are crucial in dynamic CMS situations.

2.4. Under-utilization of cells

In CMS, the underutilization of machines within cells is a significant problem that frequently arises from the lack of properly optimized scheduling and routing. [15] addressed the issue by proposing a TOPSIS-based hybrid memetic algorithm to decrease intercellular mobility and fluctuation in cell load. Although the technique offers a strong framework for optimization, complex CMS setups can cause it to struggle with computational efficiency. By modifying the size of the cells and batches, [16] presented a binary scheduling model to reduce intracellular and intercellular movements; however, the binary nature of the model could restrict its adaptability in practical applications.

[17] used a genetic algorithm for the creation of part families to balance the cell load and minimize intercellular movements. Although genetic algorithms are good at producing high-quality solutions, they frequently require a lot of processing power. They cannot scale well as system complexity increases, [18, 19]. Interestingly, most scheduling strategies in the literature do not address the need to strike the ideal balance between in-house and outsourced manufacturing in dynamic CMS.

In conclusion, the literature lacks a comprehensive method for balancing cell load regulation and the flexibility needed in demand-changing situations, even though dynamic CMS has been extensively studied in areas such as part routing, cell reconfiguration, and usage. To fill these gaps, this work suggests a mixed-integer linear programming technique that optimizes cell load regulation in the short term under dynamic cost restrictions and fluctuating demand.

3. Problem formulation

In this section, we present the *mixed-integer linear programming* (MILP) model for our target problem, which is optimizing the regulation of cell load in dynamic cell manufacturing systems. The model aims to minimize the costs of machine setups, operations, procurement, subcontracting, holding inventory, and material movements (intra-cellular and inter-cellular). In the following, we define the model's indices, parameters, decision variables, objective function, and constraints.

3.1. Assumptions

1) Each machine M has a capacity CM (in terms of production volume), and the total quantity of products that are processed on machine M must not exceed this capacity.

2) Each cell c is subject to a capacity CL, which defines the total amount of products that can be processed within that cell. The amount of product processed in a cell must not exceed this total capacity.

3) The demand for parts in each scheduling period follows a distribution that adheres to a probability law, such as the normal distribution.

4) Subcontracting services are allowed for specific products; however, the capacity of subcontractors is limited.
5) Backorders are permitted, but are limited. The initial stock (operations stock) is zero and backorders from the previous period are not allowed.
6) Product types are associated with a certain number of operations (routes) that must be carried out according to manufacturing priorities.

3.2. Parameters and variables

Below, we define the different indexes of our model (Table 1), the parameters (Table 2), and the optimization variables (Table 3).

Table 1. Definition of indices used in the model

Index	Description
p	Index for products
o	Index for operations
m	Index for machine types
f	Index for specific machines
c	Index for cells
l	Index for subcontractors
t	Index for planning periods (in minutes)

3.3. Description of intracellular and intercellular movements

This research investigates the movement of materials within and between manufacturing cells. The model focuses on tracking *work-in-process* (WIP) transfers during production, considering both intracellular (within a cell) and intercellular (between cells) movements. The model employs binary variables to indicate whether a given operation is performed on a specific machine, and equations are developed to quantify WIP transfers based on the difference in work completed on different machines. By analyzing these movements, the research aims to understand and optimize the efficiency of the manufacturing process.

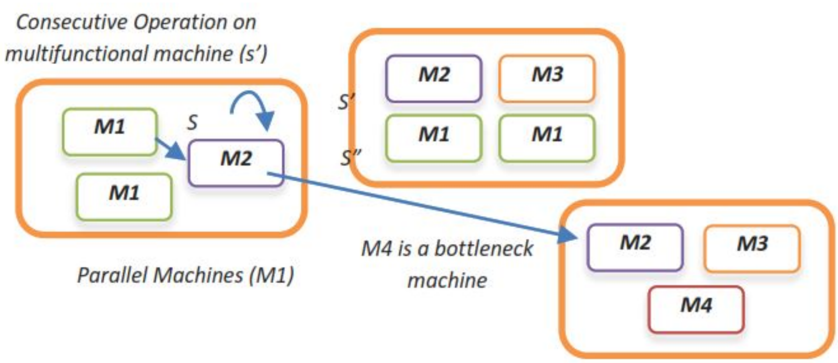

Figure 1. Intracellular and intercellular movements [8].

One of our key contributions is the linearization of the nonlinear integer programming model (NL-MIP) proposed by [8] into a *linear integer programming model* (L-MIP).

Objective function

Table 2. Parameters and their descriptions

Parameter	Description
CM_m	Capacity of machine m for product manufacturing.
$Dem_{p,t}$	Random demand for product p at time t following $N(\mu_{p,t}, \rho_{p,t})$.
$OP_{p,o,m}$	Cost of operation o performed by machine m for product p.
CL_c	Maximum cell capacity in terms of number due to cell c space limitation.
$SubCapacity_l$	Capacity of subcontractor l.
$SubCost_l$	Cost of subcontracting production to l.
INT_m	Initial number of machines of type m.
$IntrCost_p$	Intracellular movement cost for product p.
$InterCost_p$	Intercellular movement cost for product p.
$MCIM_{p,m}$	Machine incidence matrix indicating the compatibility of product p with machine m.
$HoldCost_p$	Holding cost for delayed production of product p.
$Tlot_p$	Batch size for product p.
$SetCost_m$	Setup cost for machine m.
$Pdef_{p,t}$	Cost of deferring product p production to time t.
$MCost_m$	Purchase cost of machine m.
$SalCost_m$	Sale cost of machine m.
LP	Maximum number of machines in cell c.
P	Number of products.
O	Number of operations.
M	Number of machine types.
F	Number of all machines.
C	Number of cells.
L	Number of subcontractors.
T	Number of planning periods.

Table 3. Variables and Their Descriptions

Variable	Description
$Z_{o,p,m,f,c,t}$	Binary: 1 if operation o for product p is performed by machine f of type m in cell c at time t; 0 otherwise.
$X_{o,p,m,f,c,t}$	Integer: Number of products p processed by machine f of type m in cell c at time t.
$Y_{p,l,t}$	Integer: Number of products p subcontracted to l at time t.
$B_{p,t}$	Integer: Number of products p deferred at time t.
$NAJ_{m,c,t}$	Integer: Number of machines m added to cell c at time t.
$NRE_{m,c,t}$	Integer: Number of machines m removed from cell c at time t.
$MN_{m,c,t}$	Integer: Number of machines m in cell c at time t.

The objective is to minimize variations in cell load and associated costs while balancing internal manufacturing and subcontracting. This formulation incorporates machine capacity constraints, demand satisfaction, and intra- and intercellular material movements. The problem is formulated as a Mixed Integer Linear Programming (MILP) model involving decision variables, cost components, and operational constraints to ensure the feasibility and efficiency of the manufacturing process.

The model aims to minimize the total cost, denoted by Z, which is the sum of seven distinct components, each representing a specific cost aspect. The objective function can be written as:

$$\textbf{Minimize } Z = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 \tag{1}$$

This function encapsulates the setup costs, operational costs, machine acquisition and disposal costs, subcontracting costs, holding costs for delayed orders, and intra- and intercellular movement costs. In the following, we detail the individual cost components and their significance in the formulation.

Objective Function Components

The first term, Q1, represents the setup costs for machines in different cells over the planning horizon. It accounts for the number of operations performed on each type of machine for a specific product, scaled by the setup cost per unit, and adjusted for the lot size. Mathematically, this can be expressed as:

$$Q_1 = \sum_{t=1}^{T}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{m=1}^{M}\sum_{o=1}^{O}\sum_{f=1}^{F}\left(\frac{\text{SetCost}_m}{\text{Tlot}_p}\right) X_{o,p,m,f,c,t} \tag{2}$$

The second term, Q2, corresponds to the operational costs associated with performing various operations on machines in cells. This cost is influenced by the machine-product compatibility matrix (MCIM) and the operational cost per product-machine combination (OP).

$$Q_2 = \sum_{t=1}^{T}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{m=1}^{M}\sum_{o=1}^{O}\sum_{f=1}^{F}\text{MCIM}_{p,m}\cdot \text{OP}_{p,o,m}\cdot X_{o,p,m,f,c,t} \tag{3}$$

The third term, Q3, models the cost of machine acquisition and disposal. Depending on the scenario, adding or removing machines from cells incurs costs or generates revenue. This term ensures the efficient allocation of machine resources across the manufacturing system:

$$Q_3 = \sum_{t=1}^{T}\sum_{c=1}^{C}\sum_{m=1}^{M}\left[(\text{MCost}_m\cdot \text{NAJ}_{m,c,t}) - (\text{SalCost}_m\cdot \text{NRE}_{m,c,t})\right] \tag{4}$$

Subcontracting costs, represented by Q4, account for the expenses of outsourcing production to subcontractors. This term provides flexibility in balancing internal production capabilities with external resources.

$$Q_4 = \sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{p=1}^{P}\text{SubCost}_l\cdot Y_{p,l,t} \tag{5}$$

The fifth cost component, Q5, represents the holding costs for delayed orders. It penalizes delays in fulfilling the demands of products to encourage timely production.

$$Q_5 = \sum_{t=1}^{T}\sum_{p=1}^{P}\text{HoldCost}_p\cdot B_{p,t} \tag{6}$$

Q6 captures the costs associated with the movement of materials within a cell. Intracellular movements are optimized to reduce unnecessary handling of products and components:

$$Q_6 = \sum_{t=1}^{T}\sum_{c=1}^{C}\sum_{m=1}^{M}\sum_{f=1}^{F}\sum_{p=1}^{P}\sum_{o=1}^{O}\left(\text{IntrCost}_p\cdot\left(\sum_{f=1}^{F}X_{o+1,p,m,f,c,t} - X_{o+1,p,m,f,c,t}\right)\right) \tag{7}$$

Finally, Q7 accounts for the cost of intercellular movements. These costs are typically higher than intracell movements and are minimized to enhance system efficiency.

$$Q_7 = \sum_{t=1}^{T}\sum_{c=1}^{C}\sum_{m=1}^{M}\sum_{f=1}^{F}\sum_{p=1}^{P}\sum_{o=1}^{O}\left(\text{InterCost}_p\cdot\left(\sum_{c=1}^{C}\sum_{f=1}^{F}X_{o+1,p,m,f,c,t} - \sum_{m=1}^{M}X_{o+1,p,m,f,c,t}\right)\right) \tag{8}$$

Constraints

Several constraints govern the proposed optimization model to ensure feasibility, operational efficiency, and logical coherence. These constraints address demand satisfaction, machine capacity, cellular capacity, and machine movements.

Demand Satisfaction

This ensures that production and subcontracted quantities meet or exceed the demand for product p in the planning period.

$$\sum_{c=1}^{C}\sum_{m=1}^{M}\sum_{f=1}^{F}\sum_{o=1}^{O}\left[\frac{X_{o,p,m,f,c,t}}{\sum_{m=1}^{M}\text{MCIM}_{p,m}}\right]+\sum_{l=1}^{L}Y_{p,l,t}+B_{p,t}\geq \text{Dem}_{p,t}+B_{p,t-1}\quad \forall p,t\geq 0, B_{p,0}=0 \tag{9}$$

Machine-Product Compatibility

This ensures that operations are performed only on compatible machines as defined by the machine-product compatibility matrix (MCIM).

$$X_{o,p,m,f,c,t}\cdot(1-\text{MCIM}_{p,m})\leq 0\quad \forall p,m,f,o,c,t \tag{10}$$

Machine Capacity

This constraint ensures that the total workload assigned to a machine does not exceed its capacity.

$$\sum_{p=1}^{P}\sum_{o=1}^{O}X_{o,p,m,f,c,t}\leq \text{CM}_m\quad \forall m,f,c,t \tag{11}$$

Lot Size

This ensures that the production quantity for each product does not exceed the predefined lot size.

$$X_{o,p,m,f,c,t}\leq \text{Tlot}_p\quad \forall p,m,f,o,c,t \tag{12}$$

Binary Decision Linking

This links the binary variable $Z_(o,p,m,f,c,t)$ to the production variable $X_(o,p,m,f,c,t)$ where **BigM** is a sufficiently large constant.

$$X_{o,p,m,f,c,t}\leq Z_{o,p,m,f,c,t}\cdot \text{BigM}\quad \forall p,m,f,o,c,t \tag{13}$$

Subcontracting Capacity

This ensures that the quantities subcontracted to external providers do not exceed their capacity.

$$\sum_{p=1}^{P}Y_{p,l,t}\leq \text{SubCapacity}_l\quad \forall l,t \tag{14}$$

Holding Costs for Delayed Orders

This limits the number of delayed products on the basis of a predefined threshold.

$$B_{p,t}\leq \text{Pdef}_{p,t}\quad \forall p,t \tag{15}$$

Machine Movement Balancing

This constraint ensures that at the start of the planning horizon (t=1), the number of machines of type m assigned to cell c, $MN_(m,c,t)$ is equal to the initial number of machines (INT_m) available in the system.

$$\text{MN}_{m,c,t} = \text{INT}_m \quad \forall t = 1, c \tag{16}$$

This balances each machine count after accounting for additions and removals for each cell.

$$\text{NAJ}_{m,c,t} - \text{NRE}_{m,c,t} + \text{MN}_{m,c,t-1} = \text{MN}_{m,c,t} \quad \forall m, t \geq 1, c \tag{17}$$

Cellular Capacity
This constraint enforces the minimum and maximum number of machines in each cell.

$$LP \leq \sum_{m=1}^{M} \text{MN}_{m,c,t} \leq \text{CL}_c \quad \forall c, t \tag{18}$$

Machine Utilization
This ensures that the workload assigned to the machines is proportional to their availability in the system.

$$\sum_{c=1}^{C}\sum_{f=1}^{F}\sum_{p=1}^{P}\sum_{o=1}^{O} X_{o,p,m,f,c,t} \leq \text{CM}_m \cdot \sum_{c=1}^{C} \text{MN}_{m,c,t} \quad \forall t, m \tag{19}$$

Variable Type Constraints

The following variables are defined as integers:

$$X_{opmfct}, \quad NRE_{mct}, \quad NAJ_{mct}, \quad Y_{plt}, \quad B_{pt}$$

The following variable is defined as binary:

$$Z_{opmfct}$$

These constraints ensure that production quantities, machine additions/removals, subcontracted units, and stock decisions are modeled as integer variables, while assignment variables are binary.

The above constraints and the objective function constitute the proposed *Mixed-Integer Linear Programming* (MLIP) model to optimize the regulation of cell load in dynamic manufacturing systems. The next chapter will discuss the experimental setup and present the results to evaluate the effectiveness of the model.

4. Numerical experiments

In the next section, we assess the effectiveness of our MILP model, which was implemented in the LINGO 19.0 optimization solver. For each model, we were interested in whether the optimal solution was achieved and its value, the optimal production plan, the processing time, and the number of iterations. In addition, we analyze the impact of cellular capacity on these optimal variables.

4.1. The test problems

We have considered instances taken from the literature, specifically from the work of [8]. Hence, we consider instances where four products (P1, P2, P3, and P4) are manufactured in a two-cell production shop over two periods.

Table 1 below presents all the different product data that contribute to the smooth running of the manufacturing process: demand is the demand for parts in each machine, subcontractors help balance the manufacturing process and meet deadlines, especially when their capacities are taken into account, manufacturing batch sizes and also when product carry-overs are taken into account, not forgetting related costs such as intercellular and intracellular movements, subcontractor costs per product, carry-over costs, etc.

Table 4. Machine Data

Machine	Initial Number	Capacity	Setup Cost	Machine Purchasing Cost	Machine Sale Cost
M1	5	100	100	8000	6800
M2	7	150	140	12000	10200
M3	4	75	200	7500	6000

Information about the different types of machines is also included in Table 4

Table 5 describes the production operation process routes. For example, product 1 must visit machine types 1, 2, and 3.

TABLE 5

The MCIM matrix for the experiment

Table 5. MCIM Matrix for the experiment

Products	M1	M2	M3
P1	1	1	1
P2	0	0	1
P3	0	1	1
P4	1	0	0

TABLE 6 shows the cost of each operation in the different types of machines. For example, the cost required to perform sequential operations in product 1 is 7 for the operation on machine type 1, 12 for the operation on machine type 2, and 4 for the operation on machine type 3.

TABLE 6 The operating cost of using each type of machine

Table 6. operating cost

Operation	M1	M2	M3
P1	7	12	4
P2	0	0	5
P3	0	9	4.6
P4	8	0	0

4.2. The results

We performed several small-scale experiments using the data above. The results obtained are analyzed in detail using Python data automation. All models are solved using the LINGO 19.0 optimization solver on a CoreTM I5 personal computer with a 2.60 GHz processor and 8 GB RAM.

Tables 7 and 8 below present all the different scenarios and product data that contribute to the smooth running of the manufacturing process: demand is the demand for parts in each machine, subcontractors help balance the manufacturing process and meet deadlines, especially when their capacities are taken into account, manufacturing batch sizes, and also when product carry-overs are taken into account, not forgetting related costs such as intercellular and intracellular movements, subcontractor costs per product, carry-over costs, etc.

We first generated 50 instances of problems in one scenario using the cellular capacity (LP=20; CL=30). Detailed results are provided in Table 9.

The first two columns in this table provide the two parameters of cellular capacity, LP and CL. The following two columns provide d11 and d12, the two demands for product P1 to be satisfied in Cells 1 and 2. These two values are generated randomly from the data in Table 7. The following six columns represent similar data for P2, P3 and P4.

The column “Optimal Solution” is the best solution by LINGO, and the column “Bound” is the best lower bound achieved. One can easily see that for all 50 instances, the “Optimal Solution” and the “Bound” are the same. This indicates that all models were solved to optimality, as expected. Hence, the “Optimal” column indicates “Y” for Yes. The columns “Production,” “Subcontract,” and “Reported” provided the optimal values for the planned decisions.

Table 7. Product data set for the experiment

Product	Demand	Supplier	Outsource capacity	Batch size	Allowed Backorders	Outsourcing cost (per product)	Backorder cost (per product)	Intracellular Cost$	Intercellular cost$
P1	$N(100, 12^2)$	S1	100	15	[20,0]	50	4	3	4
P2	$N(60, 8^2)$	S2	50	10	[30,0]	14	3.5	5	8
P3	$N(75, 14^2)$	S3	60	8	[45,0]	20	3	4	6
P4	$N(200, 4^2)$	S4	120	15	[60,0]	18	2.4	7	8

Table 8. Product Averages and Standard Deviations by Scenario

Scenario	P1 Average	P1 Standard Deviation	P2 Average	P2 Standard Deviation	P3 Average	P3 Standard Deviation	P4 Average	P4 Standard Deviation
SCENARIO 1	100	12	70	9	75	14	200	4
SCENARIO 2	120	14	40	6	55	12	200	4
SCENARIO 3	120	14	40	6	75	14	180	2
SCENARIO 4	90	11	70	9	55	12	200	4
SCENARIO 5	120	14	80	10	95	16	190	3
SCENARIO 6	100	12	60	8	85	15	220	6
SCENARIO 7	110	13	80	10	55	12	210	5
SCENARIO 8	120	14	60	8	65	13	220	6
SCENARIO 9	90	11	70	9	95	16	200	4
SCENARIO 10	80	10	40	6	95	16	180	2
SCENARIO 11	120	14	50	7	85	15	190	3
SCENARIO 12	110	13	80	10	65	13	220	6
SCENARIO 13	80	10	50	7	85	15	190	3
SCENARIO 14	120	14	70	9	65	13	190	3
SCENARIO 15	90	11	40	6	75	14	200	4
SCENARIO 16	120	14	70	9	95	16	180	2
SCENARIO 17	120	14	70	9	95	16	210	5
SCENARIO 18	80	10	80	10	85	15	210	5
SCENARIO 19	120	14	70	9	65	13	180	2
SCENARIO 20	80	10	50	7	85	15	190	3

Hence, for the 50 instances, no demand satisfaction has been reported. The demand was satisfied by combining in-house production and subcontracting (which, in this instance, was always the same amount, 460). The processing times were reported to be on average 0.56 seconds with a 0.04 standard deviation. On average, the number of iterations was 39.38 and 2.87 for the standard deviation. Note that our problems have 2,837 variables, including 1,312 integer ones, and 9,284 constraints for all models here.

We have then assessed the impact of cellular capacity, represented here by the two variables LP and CL. Please recall that these two parameters control the number of machines in the manufacturing system for each cell and each time. We started this assessment by solving 50 instances of the model for LP=0 and CL=30. This allows the system to have 0 and 30 machines at any given time.

The results of these second instances are presented in Table 10. All the columns have the same meaning as before. In the columns “Optimal Solution” and “Bound,” we can observe that we have negative numbers this time, even though all problems were solved optimally. The columns “Production,” “Subcontract,” and “Reported” provided the optimal values for the planned decisions. Hence, again, for the 50 instances, no demand satisfaction has been reported. The demand was satisfied through a combination of in-house production and subcontracting. This time, neither of the two quantities was fixed (as was the last time subcontracting). On average, the in-house production time was 383.78, with a standard deviation of 48.36, whereas the subcontracting quantity was 503.98, with a standard deviation of 34.09.

Table 9. Summary of optimal solutions and parameters for LP=20 and CL=30

LP	CL	d11	d12	d21	d22	d31	d32	d41	d42	Optimal Solution	Bound	Optimal	Production	Subcontract	Reported	Time	Iterations
20	30	126	133	83	77	95	62	179	189	41547.33	41547.33	Y	484	460	0	0.53	48
20	30	115	114	95	85	103	113	194	188	43351.67	43351.67	Y	547	460	0	0.62	40
20	30	128	123	61	64	92	100	188	190	41510.00	41510.00	Y	486	460	0	0.51	37
20	30	102	142	67	65	100	87	193	190	41472.33	41472.33	Y	486	460	0	0.53	35
20	30	99	108	86	60	86	99	192	190	40696.67	40696.67	Y	460	460	0	0.53	39
20	30	103	130	66	92	109	116	197	183	43297.33	43297.33	Y	568	460	0	0.52	39
20	30	143	135	80	75	121	96	185	185	44426.67	44426.67	Y	630	460	0	0.52	42
20	30	127	124	84	86	95	101	186	184	42843.67	42843.67	Y	527	460	0	0.53	38
20	30	97	137	81	80	109	120	188	187	43604.00	43604.00	Y	593	460	0	0.59	38
20	30	130	115	55	66	85	100	189	188	40978.33	40978.33	Y	468	460	0	0.53	36
20	30	97	80	74	77	120	85	185	188	40352.67	40352.67	Y	446	460	0	0.52	39
20	30	114	95	89	71	99	65	186	189	40407.00	40407.00	Y	448	460	0	0.53	39
20	30	135	114	58	84	79	95	191	193	41553.00	41553.00	Y	489	460	0	0.73	37
20	30	109	100	47	100	112	50	190	191	40075.00	40075.00	Y	439	460	0	0.71	35
20	30	112	140	66	66	112	130	188	186	43849.33	43849.33	Y	620	460	0	0.63	38
20	30	112	140	66	66	112	130	188	186	43849.33	43849.33	Y	620	460	0	0.63	38
20	30	127	106	84	75	75	109	190	189	41771.67	41771.67	Y	495	460	0	0.53	37
20	30	118	124	86	79	97	111	190	192	43121.67	43121.67	Y	547	460	0	0.63	38
20	30	109	108	100	80	61	78	191	189	40593.33	40593.33	Y	456	460	0	0.53	35
20	30	140	110	86	89	60	103	187	192	42143.67	42143.67	Y	507	460	0	0.54	38
20	30	125	140	101	78	118	115	189	188	45463.33	45463.33	Y	670	460	0	0.61	44
20	30	122	124	93	75	84	90	190	191	42184.00	42184.00	Y	509	460	0	0.54	43
20	30	110	105	73	74	106	103	188	187	41535.00	41535.00	Y	486	460	0	0.54	40
20	30	130	127	76	90	90	73	191	190	42121.00	42121.00	Y	507	460	0	0.65	40
20	30	104	123	81	91	102	99	189	191	42536.33	42536.33	Y	520	460	0	0.53	44
20	30	101	122	91	78	106	67	191	188	41449.67	41449.67	Y	484	460	0	0.54	37
20	30	114	138	80	91	111	93	195	183	43343.00	43343.00	Y	547	460	0	0.53	40
20	30	114	115	96	91	103	85	189	192	42679.00	42679.00	Y	525	460	0	0.59	38
20	30	129	122	86	77	108	94	190	193	43206.33	43206.33	Y	553	460	0	0.53	41
20	30	130	127	87	91	110	117	189	191	44839.33	44839.33	Y	630	460	0	0.54	39
20	30	122	122	83	69	102	84	187	193	41990.33	41990.33	Y	502	460	0	0.53	42
20	30	139	139	73	75	91	111	185	193	43678.00	43678.00	Y	586	460	0	0.53	40
20	30	126	128	85	87	97	68	185	189	42131.33	42131.33	Y	505	460	0	0.54	44
20	30	113	93	79	63	68	96	189	187	39786.67	39786.67	Y	428	460	0	0.53	37
20	30	117	117	74	74	101	62	191	192	40930.33	40930.33	Y	468	460	0	0.52	41
20	30	137	121	81	85	97	74	188	190	42409.00	42409.00	Y	521	460	0	0.54	42
20	30	130	115	72	71	97	90	190	185	41664.00	41664.00	Y	490	460	0	0.53	38
20	30	117	112	77	95	88	111	191	192	42606.33	42606.33	Y	523	460	0	0.55	37
20	30	122	131	71	82	88	87	188	185	41801.67	41801.67	Y	494	460	0	0.55	38
20	30	114	121	75	87	98	71	191	189	41493.33	41493.33	Y	486	460	0	0.54	41
20	30	120	110	73	71	101	76	187	185	40874.00	40874.00	Y	463	460	0	0.52	40
20	30	119	132	62	72	88	103	184	188	41717.00	41717.00	Y	498	460	0	0.54	37
20	30	102	138	70	90	102	92	193	190	42428.33	42428.33	Y	517	460	0	0.53	35
20	30	110	99	81	84	111	94	189	192	41917.00	41917.00	Y	500	460	0	0.62	38
20	30	135	82	80	87	87	101	194	188	41728.67	41728.67	Y	494	460	0	0.54	45
20	30	126	108	78	76	92	96	196	187	41864.33	41864.33	Y	499	460	0	0.54	39
20	30	93	143	92	76	59	78	187	192	40728.67	40728.67	Y	460	460	0	0.54	38
20	30	138	122	86	74	100	114	189	188	43746.33	43746.33	Y	579	460	0	0.52	45
20	30	109	99	67	72	86	74	189	189	39680.00	39680.00	Y	425	460	0	0.52	38
20	30	116	123	73	59	92	95	192	191	41318.33	41318.33	Y	481	460	0	0.59	42
Number										50	50		50	50		50	50
Minimum										39680	39680		425	460		0.51	35
Average										42105.91	42105.91		513.32	460		0.56	39.38
Median										41890.67	41890.67		499.5	460		0.54	39
Standard deviation										1289.60	1289.60		55.86	0.00		0.05	2.87
Maximum										45463.33	45463.33		670	460		0.73	48

The processing times were reported to be on average 0.65 seconds with a 0.11 standard deviation.

On average, the number of iterations was 830.74 and 1420.39 for the standard deviation.

Finally, the most crucial difference comes from the optimal value. All optimal values were positive when the cell capacity was LP = 20 and CL = 30. Here, with LP=0 and CL=30, they are negative. This suggests significant additional costs as a result of selling more machines in the latter case. In fact, with LP = 20, 22 machines were acquired and 14 sold. With LP=0, an average of 30 machines were sold.

We explored this idea further with two additional analyses.

In the first, we maintain CL at 30 and vary LP from 0 to 28 by increments of 4. For each cellular capacity scenario, 50 instances were solved. In the second, LP and CL are randomly selected, and again, for each cellular

Table 10. Summary of optimal solutions and parameters for LP=0 and CL=30

LP	CL	d11	d12	d21	d22	d31	d32	d41	d42	Optimal Solution	Bound	Optimal	Production	Subcontract	Reported	Time	Iterations
0	30	106	87	63	73	116	76	200	205	-226453	-226453	Y	455	471	0	0.58	214
0	30	91	96	75	66	57	70	195	195	-233010	-233010	Y	288	557	0	0.81	440
0	30	102	94	71	80	57	60	205	201	-228295	-228295	Y	405	465	0	0.64	1165
0	30	99	81	70	65	58	82	198	194	-233108	-233108	Y	295	552	0	0.66	415
0	30	91	87	79	66	72	71	196	194	-232927	-232927	Y	308	548	0	0.58	195
0	30	110	115	76	73	90	61	193	196	-226565	-226565	Y	438	478	0	0.77	753
0	30	114	105	66	82	103	88	202	205	-224520	-224520	Y	455	510	0	0.63	220
0	30	97	88	73	78	64	79	194	196	-228003	-228003	Y	397	474	0	0.83	2138
0	30	71	102	72	73	97	88	201	205	-226432	-226432	Y	411	498	0	0.69	223
0	30	80	114	56	53	66	71	196	205	-228697	-228697	Y	368	473	0	0.66	184
0	30	89	103	82	69	78	75	206	199	-227125	-227125	Y	424	477	0	0.72	2393
0	30	95	107	62	71	88	78	201	200	-226882	-226882	Y	416	486	0	0.73	243
0	30	98	107	66	67	68	52	201	201	-232668	-232668	Y	303	557	0	0.8	549
0	30	99	91	75	68	36	79	200	197	-228755	-228755	Y	380	467	0	0.7	609
0	30	116	83	71	77	89	92	195	200	-226282	-226282	Y	441	482	0	0.63	229
0	30	100	102	66	70	92	41	202	202	-232467	-232467	Y	330	545	0	0.6	209
0	30	98	107	80	61	76	80	198	202	-226977	-226977	Y	422	480	0	0.59	224
0	30	104	93	67	65	66	69	190	202	-232657	-232657	Y	297	559	0	0.7	501
0	30	90	92	73	64	95	67	200	203	-231960	-231960	Y	328	556	0	0.83	640
0	30	77	115	71	72	81	63	197	210	-227240	-227240	Y	396	490	0	0.53	192
0	30	124	92	68	84	78	67	201	197	-226850	-226850	Y	441	472	0	0.7	736
0	30	96	90	76	75	85	83	198	194	-227123	-227123	Y	425	476	0	0.72	882
0	30	97	81	72	80	69	58	200	199	-232960	-232960	Y	308	548	0	0.55	205
0	30	103	99	73	59	96	34	201	198	-233318	-233318	Y	343	520	0	0.56	224
0	30	114	87	88	64	74	70	202	200	-231627	-231627	Y	348	551	0	0.61	698
0	30	106	74	74	61	84	81	201	198	-232358	-232358	Y	335	544	0	0.76	216
0	30	81	97	71	77	68	65	199	198	-228485	-228485	Y	387	469	0	0.92	4097
0	30	100	93	66	72	76	86	198	198	-227330	-227330	Y	408	481	0	0.67	373
0	30	83	110	77	73	70	79	198	200	-227097	-227097	Y	398	492	0	0.55	207
0	30	105	99	62	74	71	99	197	207	-226028	-226028	Y	405	509	0	0.54	263
0	30	85	104	78	72	96	72	198	201	-226877	-226877	Y	427	479	0	0.54	223
0	30	91	94	74	79	71	78	194	205	-227303	-227303	Y	400	486	0	0.54	222
0	30	116	100	92	52	73	72	202	204	-231127	-231127	Y	353	558	0	0.60	615
0	30	107	103	57	77	80	74	199	206	-226755	-226755	Y	413	490	0	0.53	240
0	30	86	93	73	76	80	90	195	199	-227063	-227063	Y	403	489	0	0.93	8777
0	30	104	81	67	84	89	56	198	200	-232193	-232193	Y	328	551	0	0.62	644
0	30	115	82	58	80	94	88	211	202	-226205	-226205	Y	448	482	0	0.54	231
0	30	93	88	71	68	78	72	202	199	-232347	-232347	Y	314	557	0	0.59	552
0	30	109	108	66	66	74	77	192	202	-227107	-227107	Y	411	483	0	0.53	211
0	30	102	107	58	66	77	92	201	200	-226672	-226672	Y	408	495	0	0.54	242
0	30	91	93	80	73	51	69	204	203	-228340	-228340	Y	396	468	0	0.58	783
0	30	105	98	76	73	78	95	203	192	-226098	-226098	Y	430	494	0	0.95	3924
0	30	108	90	82	72	71	60	197	202	-232035	-232035	Y	328	554	0	0.6	568
0	30	98	108	72	74	52	70	197	203	-227642	-227642	Y	389	485	0	0.54	240
0	30	114	86	71	75	81	67	196	195	-231957	-231957	Y	332	553	0	0.62	660
0	30	103	93	53	67	74	77	203	201	-228188	-228188	Y	403	468	0	0.66	1912
0	30	88	108	71	84	105	64	200	194	-226469	-226469	Y	434	484	0	0.62	746
0	30	104	112	69	67	103	81	196	203	-225710	-225710	Y	442	493	0	0.53	233
0	30	99	103	67	54	42	68	205	208	-228872	-228872	Y	383	463	0	0.6	638
0	30	93	101	79	84	48	57	202	208	-227827	-227827	Y	392	480	0	0.56	239
Number										50	50		50	50	50	50	50
Minimum										-233318	-233318		288	463	0	0.53	184
Average										-228820	-228820		383.78	503.98	0	0.56	39.38
Median										-227734	-227734		397.5	489.5	0	0.62	394
Standard deviation										2636.80	2636.80		48.36	34.09	0.00	0.11	1420.39
Maximum										-224520	-224520		455	559	0	0.95	8777

capacity, 50 instances are solved. We report the results in Table 11 for the first experiment (and Table 12 for the second experiment). The two tables present the same variables and parameters as Table 9 and Table 10, adding "Machines Acquired" and "Machines Sold". The actual values in the table are averages over the 50 instances.

The main result from both tables is that the optimal value of the models is positive or negative depending on whether the number of machines acquired is higher or smaller than the number of machines sold. This further appears to be related to the difference between CL and LP and the optimal values. Therefore, we performed a regression analysis between the slack between LP and CL (CL-LP) and the optimal values. We obtain Figure 1 above. We get a coefficient of correlation of 0.68. A regression analysis shows that the difference CL-LP can explain about 46% of the optimal value variations.

Table 11. Summary of average optimal solutions and parameters for CL=30 and LP varying from 0 to 28 by 4.

LP	CL	Instances	Optimal Solution	Bound	Optimal	Production	Subcontract	Reported	Machines Acquired	Machines Sold	Time	Iterations
0	30	50	-228820	-228820	Y	383.78	503.98	0	0	30.32	0.6496	830.74
4	30	50	-193372	-193372	Y	365.02	460.42	0	0	24	0.5964	245.18
8	30	50	-140318	-140318	Y	370.32	460	0	0	16	0.5452	39.36
12	30	50	-80312	-80312	Y	429.6	460	0	6	14	0.5396	38.28
16	30	50	-21850.4	-21850.4	Y	371.06	460	0	14	14	0.5444	34.56
20	30	50	42105.91	42105.91	Y	513.32	460	0	22	14	0.5562	39.38
24	30	50	100486.7	100486.7	Y	467.9	460	0	30	14	0.5404	36.7

Table 12. Summary of average optimal solutions and parameters for randomly selected LP and CL.

LP	CL	Instances	Optimal Solution	Bound	Optimal	Production	Subcontract	Reported	Machines Acquired	Machines Sold	Time	Iterations
15	38	50	-35326.9	-35326.9	Y	428.96	460	0	12	14	0.5406	37.54
22	33	50	68148.1	68148.1	Y	370.72	460	0	26	14	0.5522	34.76
22	28	50	68188.29	68188.29	Y	366.9	460	0	26	14	0.545	35.08
25	36	50	113075.9	113075.9	Y	367.88	460	0	32	14	0.5464	33.82
14	28	50	-47668.6	-47668.6	Y	523.3	460	0	10	14	0.5522	40.32
20	29	50	40487.34	40487.34	Y	468.38	460	0	22	14	0.5440	37.28
12	39	50	-79923.8	-79923.8	Y	448.3	460	0	6	14	0.5432	37.36
22	40	50	70459.19	70459.19	Y	465.66	460	0	26	14	0.5582	36.82
21	32	50	55128.49	55128.49	Y	444.64	460	0	24	14	0.5614	36.34
13	35	50	-67766.1	-67766.1	Y	327.76	460	0	8	14	0.5876	35.08
24	26	50	99385.16	99385.16	Y	415.56	460	0	30	14	0.5514	36
16	38	50	-18907.1	-18907.1	Y	488.58	460	0	14	14	0.5478	37.62
17	31	50	-7283.17	-7283.17	Y	350.6	460	0	16	14	0.5506	35.22
15	32	50	-35117.5	-35117.5	Y	431.4	460	0	12	14	0.5466	38.14
12	29	50	-82415.5	-82415.5	Y	346.16	460	0	6	14	0.5394	34.76
19	27	50	26081.27	26081.27	Y	472.84	460	0	20	14	0.5396	39.02
18	35	50	12486.8	12486.8	Y	538.14	460	0	18	14	0.5426	38.84
23	29	50	84960.34	84960.34	Y	443.6531	460	0	28	14	0.5533	35.86
25	40	50	114290.5	114290.5	Y	405.76	460	0	32	14	0.5508	36.82
10	27	50	-112433	-112433	Y	344.08	460	0	2	14	0.5398	35.5

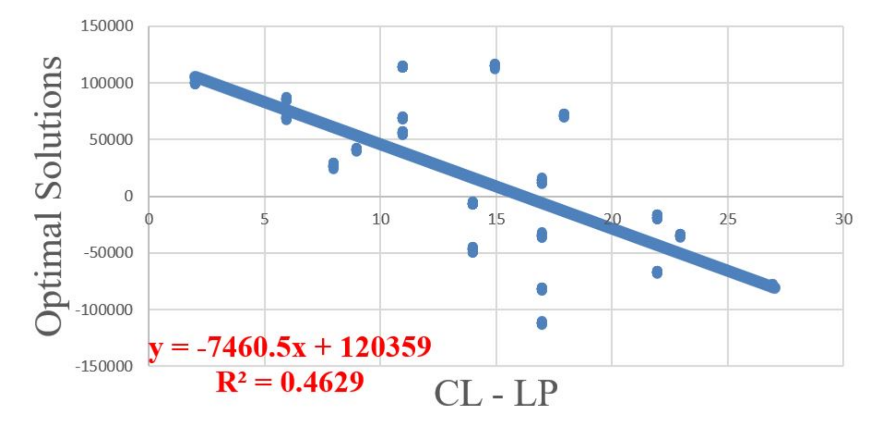

Figure 2. Regression between cell capacity and optimal solution

The main result of both tables is that the optimal value of the models is positive or negative depending on whether the number of machines acquired is higher or lower than the number of machines sold. This further appears to be related to the difference between CL and LP and the optimal values. Therefore, we performed a regression analysis between the slack between LP and CL (CL-LP) and the optimal values. We obtain Figure 2 above. We get a coefficient of correlation of 0.68. A regression analysis shows that the difference CL-LP can explain about 46% of the optimal value variations.

4.3. Additionnals results

We conducted additional experiments using a more significant scale dataset to further assess extensibility and robustness. We consider instances where ten products (P1, P2, P3, P4, P5, P6, P7, P8, P9, and P10) are manufactured in a four-cell production shop (c=4) over four periods (T=4).

The machines used were the same (see Table 4). The production process routes for the 10 products are provided in Table 13.

Table 13. MCIM Matrix for the experiment (10 products)

Produits	M1	M2	M3
P1	1	1	1
P2	0	0	1
P3	0	1	1
P4	1	0	0
P5	1	1	1
P6	0	0	1
P7	0	1	1
P8	1	0	0
P9	1	1	1
P10	0	0	1

The corresponding operating costs are shown in Table 14.

Table 14. Operations costs (10 products

Opération	M1	M2	M3
P1	7	12	4
P2	0	0	5
P3	0	9	4.6
P4	8	0	0
P5	10	14	5
P6	0	0	4
P7	0	9	3
P8	6	0	0
P9	12	7	8
P10	0	0	4.6

Finally, the product data are summarized in Table 15.

Table 15. Product data set for the experiment (10 products)

Product	Demand	Supplier	Outsource capacity	Batch size	Allowed Backorders	Outsourcing cost (per product)	Backorder cost (per product)	Intracellular Cost$	Intercellular cost$
P1	$N(100, 12^2)$	S1	100	15	[20,0]	50	4	3	4
P2	$N(60, 8^2)$	S2	50	10	[30,0]	14	3.5	5	8
P3	$N(75, 14^2)$	S3	60	8	[45,0]	20	3	4	6
P4	$N(200, 4^2)$	S4	120	15	[60,0]	18	2.4	7	8
P5	$N(190, 6^2)$	S5	60	10	[50,0]	50	5	6	4
P6	$N(110, 10^2)$	S6	70	15	[30,0]	30	3.4	3	6
P7	$N(50, 4^2)$	S7	120	8	[70,0]	25	6	8	4
P8	$N(90, 7^2)$	S8	100	10	[20,0]	55	3	4	8
P9	$N(210, 5^2)$	S9	50	15	[10,0]	60	4	5	5
P10	$N(130, 8^2)$	S10	80	10	[25,0]	40	2.5	9	7

We proceeded as previously. All the instances solved here were generated following the 20 scenarios proposed in Table 16. In the first analysis, we maintain CL at 30 and vary LP from 0 to 28 by increments of 4. For each

Table 16. Products averages and standard deviations (10 products)

Scenario	P1 A	P1 SD	P2 A	P2 SD	P3 Av	P3 SD	P4 A	P4 SD	P5 A	P5 SD	P6 A	P6 SD	P7 A	P7 SD	P8 A	P8 SD	P9 A	P9 SD	P10 A	P10 SD
SCENARIO 1	110	13	60	8	85	15	180	2	110	10	200	4	40	3	70	5	220	6	140	9
SCENARIO 2	120	14	50	7	75	14	180	2	100	9	180	2	30	2	100	8	210	5	150	10
SCENARIO 3	120	14	50	7	85	15	180	2	130	12	190	3	40	3	110	9	200	4	120	7
SCENARIO 4	110	13	70	9	95	16	220	6	130	12	220	6	60	5	110	9	230	7	150	10
SCENARIO 5	120	14	70	9	55	12	190	3	110	10	220	6	40	3	80	6	220	6	110	6
SCENARIO 6	90	11	50	7	65	13	210	5	100	9	210	5	50	4	100	8	220	6	150	10
SCENARIO 7	100	12	70	9	75	14	190	3	110	10	210	5	40	3	80	6	210	5	110	6
SCENARIO 8	80	10	60	8	65	13	220	6	130	12	200	4	30	2	110	9	230	7	120	7
SCENARIO 9	100	12	70	9	65	13	220	6	90	8	220	6	40	3	90	7	210	5	150	10
SCENARIO 10	100	12	40	6	95	16	180	2	130	12	200	4	40	3	90	7	220	6	120	7
SCENARIO 11	90	11	60	8	95	16	180	2	100	9	210	5	40	3	80	6	210	5	150	10
SCENARIO 12	90	11	40	6	65	13	210	5	130	12	220	6	70	6	70	5	230	7	120	7
SCENARIO 13	90	11	70	9	85	15	220	6	130	12	190	3	70	6	70	5	210	5	120	7
SCENARIO 14	80	10	70	9	55	12	190	3	90	8	200	4	30	2	90	7	200	4	110	6
SCENARIO 15	110	13	50	7	75	14	220	6	130	12	210	5	60	5	90	7	230	7	130	8
SCENARIO 16	90	11	80	10	85	15	190	3	100	9	200	4	50	4	80	6	200	4	120	7
SCENARIO 17	100	12	50	7	85	15	190	3	90	8	190	3	70	6	80	6	200	4	130	8
SCENARIO 18	80	10	60	8	75	14	190	3	130	12	210	5	40	3	90	7	210	5	150	10
SCENARIO 19	100	12	50	7	55	12	220	6	130	12	180	2	60	5	100	8	190	3	150	10
SCENARIO 20	110	13	70	9	85	15	190	3	100	9	200	4	40	3	110	9	230	7	140	9

cellular capacity scenario, 50 instances were solved. In the second one, LP and CL are randomly selected, and again, for each cellular capacity, 50 instances are solved. We report the results in Table 17 for the first experiment (and Table 18 for the second experiment). The two tables present the same variables and parameters as Table 9 and Table 10, adding “Machines Acquired” and “Machines Sold”. The actual values in the table are averages over the 50 instances.

Table 17. Summary of average optimal solutions and parameters for CL=30 and LP varying from 0 to 28 by 4 (10 products)

LP	CL	Instances	Optimal Solution	Bound	Optimal	Production	Subcontract	Reported	Machines Acquired	Machines Sold	Time	Iterations
0	30	50	-317126	-317126	Y	3208.76	2266.88	330	0	54.16	51.276	45752.3
4	30	50	-301381	-301381	Y	2676.74	1989.98	330	0	48	46.5906	13145.9
8	30	50	-193026	-193026	Y	2886.36	1985.42	330	0	32	37.5816	592.64
12	30	50	-50737.1	-50737.1	Y	3601.56	2189.36	330	12	28	37.6204	213.26
16	30	50	43344.19	43344.19	Y	2851.82	1989.18	330	28	28	36.7662	204.36
20	30	50	168915.7	168915.7	Y	2906.2	2034.7	330	44	28	36.1184	205.74
24	30	50	345051.9	345051.9	Y	2902.04	1979.76	330	60	28	36.3879	202.42

Table 18. Summary of average optimal solutions and parameters for randomly selected LP and CL (10 products).

LP	CL	Instances	Optimal Solution	Bound	Optimal	Production	Subcontract	Reported	Machines Acquired	Machines Sold	Time	Iterations
15	38	50	14391.94	14391.94	Y	2892.8	1965.56	330	24	28	36.6352	212.02
22	33	50	222287.7	222287.7	Y	2816.84	1945.74	330	52	28	36.5026	204.28
22	28	50	225262.2	225262.2	Y	2918.8	1995.24	330	52	28	36.9786	203.56
25	36	50	343070.3	343070.3	Y	3520.68	2237.2	330	64	28	41.535	215.7
14	28	50	-14397.5	-14397.5	Y	2537.96	2133	330	20	28	38.603	211.26
20	29	50	171814	171814	Y	2680.44	2166.8	330	44	28	38.603	211.26
12	39	50	-76258.1	-76258.1	Y	2487.74	2113.3	330	12	28	37.0156	211.5
22	40	50	231457.2	231457.2	Y	2707.68	2157.52	330	52	28	37.1694	206.06
21	32	50	203238.3	203238.3	Y	2751.26	2168.82	330	48	28	37.5128	208.8
13	35	50	-44169.6	-44169.6	Y	2643.06	2070.64	330	16	28	37.1756	215.56
24	26	50	286771.5	286771.5	Y	2527.7	2137.4	330	60	28	36.6676	209.76
16	38	50	50971.82	50971.82	Y	2692.94	2150.34	330	28	28	36.3344	207.92
17	31	50	81706.49	81706.49	Y	2756.54	2140.62	330	32	28	35.533	205.26
15	32	50	2696.561	2696.561	Y	2165.58	2023.98	330	24	28	35.3286	208.28
12	29	50	-61129.1	-61129.1	Y	2983.12	2183.94	330	12	28	35.1292	219.34
19	27	50	133295	133295	Y	2433.36	2125.98	330	40	28	35.1368	206.7
18	35	50	101642.9	101642.9	Y	2436.86	2080.2	330	36	28	34.8606	199.22
23	29	50	256278.5	256278.5	Y	2505.84	2141.52	330	56	28	36.1012	208.74
25	40	50	318400.6	318400.6	Y	2590.58	2155.4	330	64	28	35.6846	205.26
10	27	50	-124592	-124592	Y	2887.86	2158.3	330	4	28	35.7704	219.72

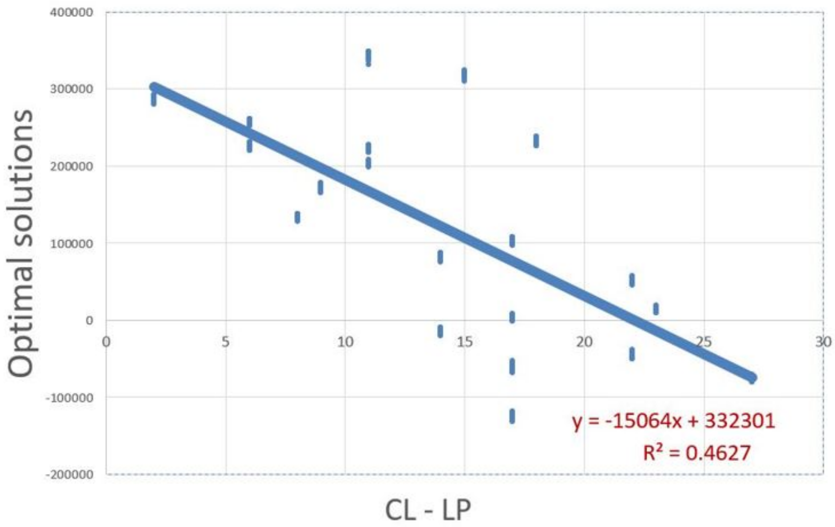

Figure 3. Regression between cell capacity and optimal solution (10 products)

The model performs satisfactorily, as shown by the results in Table 17 and Table 18. The previous main result is confirmed: the optimal value of the models is positive or negative depending on whether the number of machines acquired is higher or smaller than the number of machines sold. We also confirmed that there was a correlation between the slack between CL and LP and the optimal values. Please refer to Figure 2 - Regression between cell capacity and optimal solution (10 products). We get a coefficient of correlation of 0.67. A regression analysis shows that the slack CL-LP can explain about 46% of the optimal value variations.

In summary, using a small computational analysis, we have shown that the proposed MILP model effectively generates feasible production plans. We have also demonstrated that cell capacity parameters (LP, CL) impact the computations of optimal solutions.

This work has limitations that prevent the conclusion from being considered. First, we have to analyze other parameters that impact solutions. For example, the structures of the costs of each production option and the level of demand are parameters whose impact should be analyzed. Then, we have the size of the test problems. Instances with more than four products, two periods, and two cells shall be tested.

5. Conclusion

We introduce a mixed-integer linear programming (MILP) formulation of cellular manufacturing systems to minimize variation in cell load and associated costs while balancing internal manufacturing and subcontracting. The model comes from a linearization of the model by [8, 20].

Using LINGO 19.0, and a set of medium-scale instances, we perform extensive tests on problems with random demands. The results show the effectiveness of the model and the optimal solutions. The calculation of solutions is effective with very low processing times. In addition, we explore the impact of cell capacity parameters on optimal values. We have identified limitations in the above work that need to be addressed to enhance the practical relevance of this analysis.

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