STATISTICS, OPTIMIZATION AND INFORMATION COMPUTING
Stat., Optim. Inf. Comput., Vol. **14**, September 2025, pp 1458–1472.
Published online in International Academic Press (www.IAPress.org)

A New Mixed-Line Programming Approach to the Problem of Multimodal Urban Transit

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Abstract Multimodal urban transportation offers an efficient and reliable solution for urban mobility. This paper proposes a novel mathematical formulation, based on Mixed Integer Linear Programming (MILP), specifically addressing the optimization of formal public transportation modes within urban settings. Unlike existing models, our approach focuses exclusively on the formal transport sector while incorporating relevant operational constraints. The study begins with a concise review of the literature on optimization and organizational challenges in multimodal urban transport. Computational experiments are performed using an optimization solver to evaluate the performance and effectiveness of the proposed model.

Keywords Multimodality, Linear programming, integer programming, urban transportation, Mathematical model, combinatorial optimization, MILP.

DOI: 10.19139/soic-2310-5070-2158

1. Introduction

Multimodal urban transportation, defined as the integration of two or more urban transport modes to facilitate the movement of passengers or goods from origin to destination [1], has been the subject of significant research exploring optimization techniques to address transit challenges [2]. This study focuses on proposing a new **Mixed Integer Linear Programming (MILP)** formulation tailored exclusively to formal urban transport modes.

Our work specifically targets the urban transport landscape of Abidjan, Côte d'Ivoire, which, like many major African cities, features a dual transport system composed of formal and informal actors. The **formal sector** includes legally established operators such as the *Société de Transport Lagunaire (STL)*, the *Compagnie Ivoirienne de Transport Lagunaire (AQUALINES)*, and the *Société des Transports Abidjanais (SOTRA)*. These operators are characterized by predefined parking points and structured routes. In contrast, the **informal sector** comprises flexible, unregulated operators using *Pinasses* on the lagoon and minibuses like *Gbaka* and *Wôrô Wôrô* for road transport, often without fixed parking stops or schedules.

In this study, we aim to **minimize user travel time** by focusing exclusively on formal urban transport modes. The exclusion of informal modes allows us to formulate a well-defined, structured optimization problem based on the characteristics of formal transport systems. Accurate and timely arrival information is pivotal for effective trip planning, reducing stoppage times, and optimizing route choices. To achieve this, we propose a novel mathematical model that integrates various formal transport modes into a single framework.

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ISSN 2310-5070 (online) ISSN 2311-004X (print)

Our approach aligns with existing works on multimodal transport and combinatorial optimization [3, 4, 5]. However, unlike previous studies, which often address both sectors simultaneously or focus predominantly on informal modes, our contribution lies in the development of a robust **MILP formulation** that explicitly addresses the formal urban transport sector.

This model serves as a foundation for a structured and efficient multimodal transport system, paving the way for future extensions that incorporate the informal sector to deliver a comprehensive urban transport solution.

2. Literature review

A variety of studies have addressed challenges and proposed solutions for multimodal transportation systems, highlighting different optimization techniques and modeling approaches. Below, we critically analyze some of the key contributions in this field.

Drummond et al. [6] focus on the organizational challenges in multimodal air freight systems, with an emphasis on supply and demand dynamics. Their work provides valuable insights into the complexities of coordinating freight operations, but the scope remains limited to air freight, leaving out broader multimodal systems, such as those involving urban transport.

Pawel et al. [7] examine supply chain optimization from the perspective of multimodal logistics providers, employing mixed linear programming. While their study offers a robust mathematical framework for supply chain optimization, it largely targets industrial logistics, making its direct application to urban passenger transportation less evident.

Mnif et al. [8] propose a multi-objective formulation for planning multimodal transportation networks. Their work stands out by integrating multiple objectives into the optimization process, addressing both cost and operational efficiency. However, the lack of a focus on sector-specific constraints, such as those in urban transport, limits its applicability in this context.

Behiri et al. [9] introduce a Mixed Integer Linear Programming (MILP) model to minimize waiting time for daily deliveries via urban rail infrastructure. This contribution is particularly relevant to urban transportation, as it demonstrates the potential for optimization techniques to improve efficiency in rail-based multimodal systems. However, the study does not extend its scope to other modes of transport, such as buses or taxis.

Gonzalez et al. [10] propose a data-driven approach for evaluating multimodal public transport systems. Their methodology highlights the importance of real-time data integration and analysis for system optimization. While innovative, the study primarily focuses on performance evaluation rather than proposing new mathematical models.

AbuMonshar et al. [11] tackle the vehicle routing problem using a multi-objective optimization approach. Their work emphasizes route efficiency and cost minimization, offering valuable insights into logistics optimization. However, the study focuses more on routing problems than on the broader challenges of multimodal urban transport systems.

Finally, Mfenjou et al. [12] propose an intelligent transport system leveraging telecommunications technologies. While their approach introduces innovative technological solutions, it lacks a detailed mathematical modeling perspective, which limits its contribution to operational research methodologies for multimodal transport.

Zhu et al. [14] present an optimization model for ecological multimodal transport, incorporating order consolidation in a context of uncertainties. An enhanced genetic algorithm is employed to minimize costs and carbon emissions, albeit with a slight increase in transportation time. The findings highlight the significant benefits of consolidation in complex scenarios involving multiple origins and destinations. This approach proves particularly advantageous for customers sensitive to costs and emissions while enhancing transport efficiency. It thus contributes to achieving objectives of sustainability and operational effectiveness.

Moshebah et al. [15] propose an approach based on the concept of Max–Min fairness to enhance the performance of multimodal transportation networks. Drawing on advanced mathematical models, the authors aim to efficiently balance resources and constraints, thereby ensuring an equitable allocation of capacities across various modes of transport. The findings demonstrate that this method optimizes overall performance while reducing disparities among users, thus contributing to the development of a more sustainable and equitable multimodal transportation system.
While progress has been made in multimodal transportation optimization, significant gaps remain in the mathematical modeling of formal urban transport systems. Robust Mixed Integer Linear Programming (MILP) formulations, essential for addressing the complexity and constraints of these systems, are still underexplored.

This study contributes to filling this gap by proposing a novel MILP-based formulation tailored to formal transport modes, advancing mathematical modeling in this field and addressing operational realities specific to urban environments.

3. Problem Description

Optimizing multimodal urban transport systems is crucial to addressing the growing mobility challenges in developing countries. This study focuses specifically on the formal aspect of multimodal urban transport systems, using the city of Abidjan (Côte d'Ivoire) as a representative case study. The formal transport sector in Abidjan comprises structured operators such as the *Société de Transport Lagunaire (STL)*, the *Compagnie Ivoirienne de Transport Lagunaire (AQUALINES)*, and the *Société des Transports Abidjanais (SOTRA)*. These operators provide a robust foundation for exploring strategies to enhance transit efficiency and improve the user experience.

In this context, we model a scenario where a user seeks to travel from an origin point O to a destination point D within an urban transport network. To reach their destination, the user can choose among different transport modes, categorized into two main groups:

- **Road transport:** Includes formal terrestrial options such as EXPRESS, NAVETTE, WIBUS, and MONBUS.
- **Lagoon transport:** Represented by a single option, MONBATO.

The primary objective is to minimize the user's total travel time from O to D. This involves determining an optimal combination of transport modes, routes, and transitions at transfer points, while adhering to the operational constraints of the network.

By focusing exclusively on formal transport modes, this study aims to develop a rigorous and well-structured mathematical formulation. This approach is particularly suited to the requirements of urban transport systems in developing contexts, where efficient planning and integration of formal resources are critical priorities.

4. Our Proposal

Our model is based on the principles of Mixed Integer Linear Programming (MILP) to optimize formal public transportation modes while considering relevant operational constraints.

We begin our modeling process by defining the assumptions that govern the system. We then detail the key components, parameters, and decision variables used in the formulation. Finally, we present the objective function and associated constraints that aim to improve the efficiency and organization of formal multimodal transport in urban settings.

4.1. Model Assumptions

Here, we outline the key assumptions underlying the proposed model to optimize formal public transportation in a multimodal urban setting:

1. Users are assumed to opt for multimodal transportation to complete their journeys.
2. At each stop, only one mode of transport and one route can be selected, even if multiple routes and modes are available.
3. A mode of transport can be changed only once at a specific stop on a given date.
4. Each trip consists of traversing a predefined number of stops within a single journey.
5. Users are allowed to switch modes of transport multiple times to reach their destination. The maximum allowable number of mode changes is denoted by the variable $Change_{Max}$.
6. If a user arrives at a stop before the scheduled departure time of the next mode of transport, they must wait. This waiting time is explicitly represented in the model.

4.2. Model Descriptions

Urban multimodal transportation involves a structured combination of various transport modes to facilitate efficient travel. Below, we provide key definitions relevant to the formal public transport system, which form the foundation of our proposed model.

- **Stop:** A designated point where buses are stationed to serve passengers.
- **Mode:** Different urban transportation options, including EXPRESS, NAVETTE, WIBUS, MONBUS, and MONBATO.
- **Segment:** The route between two stops.
- **Trip:** The journey undertaken by a user on a specific mode of transport. A trip encompasses traversing several stop segments.
- **Itinerary:** The aggregate of trips undertaken by a user, incorporating at least one mode of transport and up to the maximum allowable modes, from the origin to the destination.

4.3. Model parameters

Symbol	Description
N	Set of stops.
K	Set of modes of transport.
o, d	Point of departure and point of arrival.
$\text{Change}_{\text{Max}}$	Maximum number of mode changes allowed.
$t^k_{i,j}$	Time spent in the vehicle on the journey from stop i to stop j using mode k.
$v_i^{k,k'}$	Waiting time for mode change from k to k' at stop i.

4.4. Optimization Variables

$$X^k_{i,j} = \begin{cases} 1 & \text{if our passenger uses mode } k \text{ from stop } i \text{ to stop } j \\ 0 & \text{if not} \end{cases} \tag{1}$$

$$Y_i^{k,l} = \begin{cases} 1 & \text{if our passenger has changed from mode } k \text{ to } l \text{ at stop } i \\ 0 & \text{if not} \end{cases} \tag{2}$$

4.5. Objective function

Our model aims to minimize the user's total travel time, as expressed by equation (3)

$$\sum_{i \in N} \sum_{j \in N} \sum_{k \in K} X_{i,j}^k * t_{i,j}^k + \sum_{i \in N} \sum_{k \in K} \sum_{l \in K} Y_i^{k,l} * v_i^{k,l} \tag{3}$$

The objective function comprises two components. The first component accounts for the time spent in each mode of public transport, while the second component represents the time spent at stops awaiting a new mode of transport to proceed with the journey.

4.6. Constraints

The mathematical model incorporates several constraints, outlined below:

4.6.1. Single Mode Transportation by Segment

$$\sum_{k \in K} X_{i,j}^k \leq 1 \quad \forall i, j \in N * N \tag{4}$$

Constraint (4) ensures that a user can select a maximum of one mode of transport for a given route segment between stops i and j.

4.6.2. Changing Modes

$$\sum_{k \in K} X_{i,j}^k + \sum_{k \in K} X_{j,h}^k \leq 2 \quad \forall i, j, h \in NNN \tag{5}$$

Constraint (5) models the scenario where a user arriving at stop j can continue the journey with the same mode of transport or switch to another mode.

4.6.3. Flow Conservation Constraints (6), (7), and (8) represent flow conservation on the route:

$$\sum_{j \in N} \sum_{k \in K} X_{o,j}^k = 1 \tag{6}$$

$$\sum_{i \in N} \sum_{k \in K} X_{i,j}^k = \sum_{i \in N} \sum_{k \in K} X_{j,i}^k \quad \forall j \in N \tag{7}$$

$$\sum_{j \in N} \sum_{k \in K} X_{j,d}^k = 1 \quad \forall i, j \in N * N \tag{8}$$

Constraint (6) ensures that the user enters the network from the origin route (point of departure). Constraint (7) maintains user continuity within urban transportation, considering potential mode changes at stops. Constraint (8) guides the user towards the exit (destination), represented in the model by index d.

4.6.4. *Capacity constraints*

$$\sum_{i \in N} \sum_{k \in K} \sum_{k' \in K} Y_i^{k,k'} \leq Change_{Max} \tag{9}$$

The constraint represented by equation (9) is the capacity constraint, which limits the number of mode changes in the model, denoted by $Change_{Max}$.

4.6.5. *Binding constraint*

$$\sum_{i \in N} X_{i,j}^k + \sum_{h \in N} X_{j,h}^{k'} \leq 1 + Y_j^{k,k'} \qquad \forall j \in N, \forall k, k' \in K * K; k \neq k' \tag{10}$$

The constraint represented by equation (10) is the one that establishes the dependency between the variables Y and X. Specifically, it activates a waiting time at point j if the user has traversed through point j.

5. Model Analysis

The proposed model stands out as a robust approach for optimizing multimodal urban transport systems, particularly in the context of developing countries. It relies on a Mixed Integer Linear Programming (MILP) formulation, offering a structured and rigorous method to address the challenges associated with multimodal transport optimization. Below are the key aspects and strengths of the model:

5.1. MILP Formulation and Flow Constraints

At the core of the model lies its MILP formulation, which is particularly suited for managing the complexity and combinatorial nature of multimodal transport optimization. By incorporating flow conservation constraints, the model ensures consistent continuity within the transport network. These constraints not only simplify the mathematical structure but also enhance computational efficiency by narrowing the search space to feasible and logical paths.

- **Flow Conservation:** The model integrates flow conservation at the origin, intermediate stops, and destination. This ensures that the user enters, traverses, and exits the network coherently, reflecting the real dynamics of transport.
- **Simplification Through Structure:** The use of flow constraints simplifies the problem formulation, making it both computationally manageable and conceptually intuitive. This simplification enables scalability while maintaining flexibility to handle complex multimodal scenarios.

5.2. Applicability to Multimodal Transport Systems

One of the key strengths of the model is its general applicability to a wide range of multimodal transport networks. While the case study focuses on Abidjan, Côte d'Ivoire, the formulation is inherently adaptable to diverse urban contexts and transport systems. Its main features include:

- **Flexibility for Various Transport Modes:** The model incorporates different types of transport modes, including formal road and lagoon options. This flexibility ensures its relevance in varied urban geographies.
- **Scalability for Larger Networks:** By limiting the number of mode changes and optimizing transitions, the model can handle medium to large-scale networks without significant loss of computational efficiency.
- **Structured and Formal Design:** The exclusive focus on formal transport modes enables the development of a well-defined and reliable optimization framework. This focus aligns with the need for structured solutions in developing cities.

5.3. Simplicity and Practical Relevance

The model achieves a balance between mathematical rigor and practical applicability. Its simple design, made possible through clearly defined parameters, variables, and constraints, ensures ease of implementation. At the same time, the integration of waiting times, mode transition limits, and operational constraints enhances its relevance to real-world scenarios.

- **User-Centric Objective:** The focus on minimizing total travel time reflects a user-centric perspective, aligned with the primary goal of improving transit efficiency and user experience.
- **Operational Feasibility:** The integration of realistic assumptions, such as mode change limits and predefined stops, ensures that the model captures the operational characteristics of formal transport systems.

5.4. Potential for Extensions

Although the model is currently designed for formal transport systems, its modular design provides a solid foundation for future improvements. For instance:

- **Integration of Informal Modes:** Future versions could extend the model to include informal transport options, increasing its scope and applicability to mixed systems.
- **Advanced Optimization Techniques:** The use of techniques such as cut generation or bound tightening could further improve computational performance, enabling the model to handle even larger networks.

In summary, the proposed MILP-based model provides a versatile, scalable, and computationally efficient framework for optimizing multimodal transport systems. By leveraging flow constraints, the model simplifies the problem while maintaining flexibility and applicability across diverse contexts. This simplicity, combined with its rigorous mathematical foundation, makes it a valuable tool for urban transport planning, particularly in developing cities aiming to enhance the efficiency of their formal transport systems.

6. Experimental mode and relative data

6.1. Experimental mode

Our Mixed-Integer Linear Programming (MILP) model, aimed at minimizing the user's travel time from departure to destination, has been successfully implemented and solved using version 20.0 of the LINGO optimization solver. LINGO is a robust optimization modeling software renowned for its ability to construct and solve mathematical optimization models efficiently. It offers a comprehensive set of features, including solvers for linear, nonlinear, quadratic, integer, and stochastic optimization problems [13]. We selected LINGO due to its availability as a free software compared to other paid solvers, making it accessible for our project. The model was executed on a machine equipped with a Core(TM) i7-8650U CPU running at 2.11 GHz and 16 GB of RAM. The multimodal nature of the model is evident through the integration of various road transport modes and a lagoon transport mode.

6.2. Relative data

This section of our research on public transportation in urban environments utilizes data from the Société des Transports Abidjanais (SOTRA), a major player in urban transportation in Abidjan. This collaboration was made possible through a confidentiality agreement with the Information and Applications Research Team (ERIA), granting us access to SOTRA's data. This data, covering the period from January to August 2023, has been crucial for a thorough analysis of the urban transportation sector, providing a concrete understanding of the operations of an organization active in this field. This access has proven essential to our goal of studying multimodal transportation in urban environments—an analysis that cannot be completed with publicly available data.

The data used to test our model reflects the real and operational functioning of SOTRA. We identified five different modes of transportation within the formal sector, each with its own set of stops distributed throughout the city. Each bus line is associated with a specific number of stops, which varies from line to line. It is also important to note that the distances between two stops are not uniform. Additionally, the maximum speed allowed in the urban environment is limited to 60 km/h.

We obtained legal authorization to use SOTRA's data to test our model. This authorization allowed us to analyze the data and extract essential characteristics necessary to calibrate our data generator.

We designed a Python-based instance generator to obtain the following real-world parameters:
$Time_mode$: Time spent by a user on a mode of transportation during their journey. $Time_wait$: Time spent by the user at the stop waiting for the bus. $Number_mode$: Number of varieties of transportation modes available. $Number_change$: Number of possible changes a user can make within the transportation network. $Number_stop$: Number of stops a user can utilize for their journey.

To calibrate our generator, we used value intervals, as well as averages and standard deviations from data provided by SOTRA. Parameter values are generated randomly while respecting the following limits: - $Time_mode$: [2; 15] minutes - $Time_waiting$: [0; 30] minutes - $Number_stops$: [5; 24] stops - $Number_changes$: [3; 10] changes - $Number_modes$: A fixed value of 5 transportation modes. - Standard deviations - Averages

Figure 1. Summary of instances

#Stops	#Total var.	#Bin. Var.	#Constraints	#Arcs	#Inst. (change max=5)	#Inst. (change max=10)
5	350	250	308	[60,90]	10	10
10	1250	750	1463	[269,405]	10	10
15	2625	1500	4218	[637,947]	10	10
20	4500	2500	9323	[1137,1709]	10	10
24	6360	3480	15603	[1663,2487]	10	10

Figure 1 provides a summary of data for different instances of a model. Here's an analysis and comments on each column:

- **#Stops**: Indicates the number of stops in each instance of the model.
- **#Total var.**: Represents the total number of variables in the model.
- **#Bin. Var.**: Indicates the number of binary variables in the model.
- **#Constraints**: Gives the total number of constraints in the model.
- **#Arcs**: Indicates the number of arcs in each instance, with a specified interval [min, max].
- **#Inst. (change max=5)**: Represents the number of instances of the model with a maximum number of changes equal to 5.
- **#Inst. (change max=10)**: Indicates the number of instances of the model with a maximum number of changes equal to 10.

The dataset provides information about the complexity and size of different instances of the model. Variations in each column reflect different configurations of the instances, including the number of stops, the number of variables and constraints, as well as the number of arcs in the model. The presence of instances for two different values of the maximum number of changes allows for comparing the model's performance in different contexts.

6.3. Data validation

To validate our model, we employed a rigorous approach using a carefully selected test dataset where we could determine the optimal values. We began by generating a random dataset using appropriate statistical methods, ensuring that the distributions were representative of real-world conditions. Next, we performed preprocessing steps, such as normalization and data cleaning, to ensure the quality and relevance of the information used. By applying established performance metrics, we evaluated our model on this test set, allowing us to compare the obtained results with predefined optimal values. This validation not only confirmed the robustness of our approach but also laid the groundwork for future studies, including the exploration of real-world data, such as that from Abidjan's transportation network, for even more accurate and meaningful results.

7. Results Analysis

The results obtained from solving the proposed MILP model are analyzed in this section. We focus on the relationship between processing time and key variables such as the number of stops, active arcs, maximum mode changes, and forbidden arcs percentage. The analysis highlights trends and provides insights into model performance under varying levels of complexity. As can be seen in the figure 2, the process time of the optimization model exhibits a clear trend based on several key parameters: the number of stops, percentage of active arcs, maximum mode changes, number of binary variables, and number of constraints.

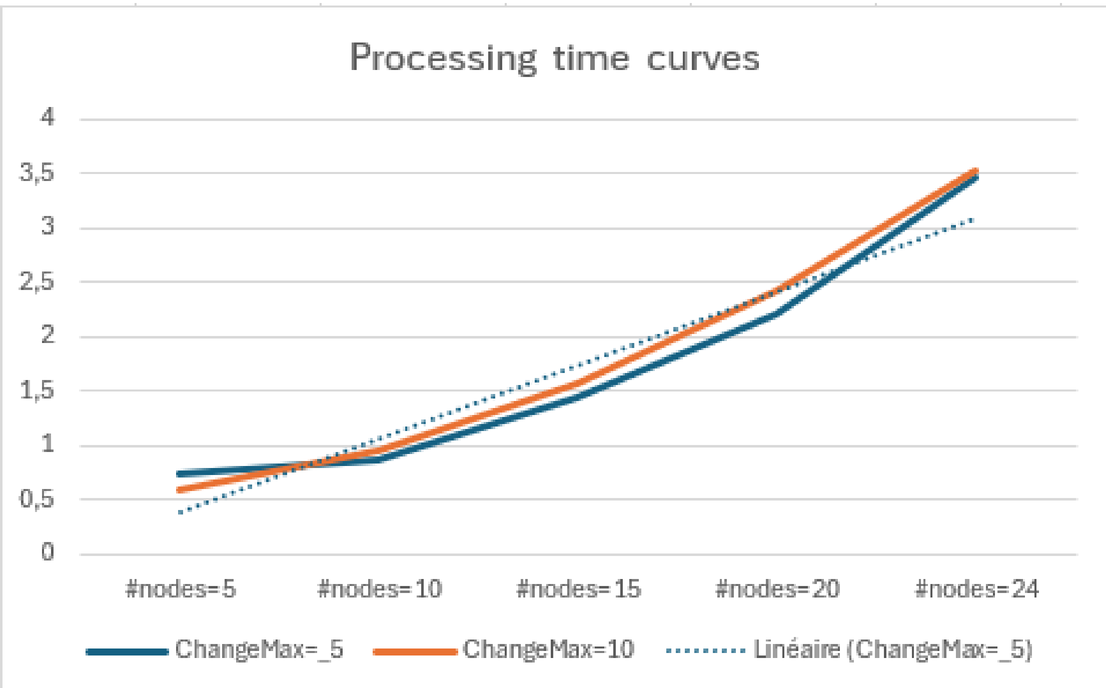

Figure 2. Processing time curves

7.1. Effect of Model Complexity

The process time increases as the model complexity grows. Specifically, instances with a higher number of stops, active arcs, and maximum mode changes tend to have longer computation times. This is because a more complex model requires the solver to explore a larger search space, leading to increased computational overhead.
The number of binary variables and constraints also influences the process time, with larger counts resulting in

longer computation times. This is due to the increased computational burden on the solver when handling a greater number of variables and constraints.

7.2. *Influence of Number of Stops and Maximum Mode Changes*

Observing the results, it is evident that the average process time increases with the number of nodes for each value of $Change_{Max}$. This trend is more pronounced when $Change_{Max}$ is set to 10. It suggests that for a higher number of allowed changes, the model requires more computational time to solve the problem as the complexity induced by the number of nodes increases. However, for $Change_{Max} = 5$, while the average process time also increases with the number of nodes, this increase appears to be slightly less significant compared to $Change_{Max} = 10$.

In general, instances with a higher number of stops tend to have longer average process times, regardless of the maximum mode changes. This suggests that the complexity of the optimization problem increases with more stops, leading to longer computational times.

These findings highlight the importance of considering both the number of stops and the maximum mode changes when optimizing multimodal transit problems. Additionally, they suggest that the optimal choice of $Change_{Max}$ may vary depending on the specific characteristics of the problem instance.

7.3. *Processing Time Analysis*

The table summarizes the results for instances across different levels of complexity. The total number of stops, active arcs, and allowable mode changes have a direct influence on the processing time.

Table 1. Summary of Processing Time Based on Instance Parameters

Instance Family	#Stops	#Modes	#Change_max	%Forbidden_Arcs	#Arcs_Active	Process_Time (s)
1	5	5	5	10	90	0.88
2	5	5	10	15	85	0.61
3	10	5	5	69	381	0.82
4	10	5	10	45	405	1.12
5	15	5	5	103	947	0.99
6	15	10	10	158	892	1.73
7	20	5	5	191	1709	1.54
8	20	10	10	286	1614	2.65
9	24	5	5	273	2487	1.62
10	24	10	10	414	2346	3.55

The data reveals the following trends:

- Processing time increases with the number of stops (#Stops) and active arcs (#Arcs_Active). Larger instances with higher arcs require more computational resources.
- Higher maximum allowable mode changes (#Change_max) increase the solution space, leading to longer solving times.
- A higher percentage of forbidden arcs reduces the solution space but increases the constraint tightness, which may lead to fluctuating processing times.

From Figure 3, it is evident that the processing time exhibits an approximately linear increase as the number of stops and active arcs grows. This trend underscores the scalability of the proposed model for medium-sized instances.

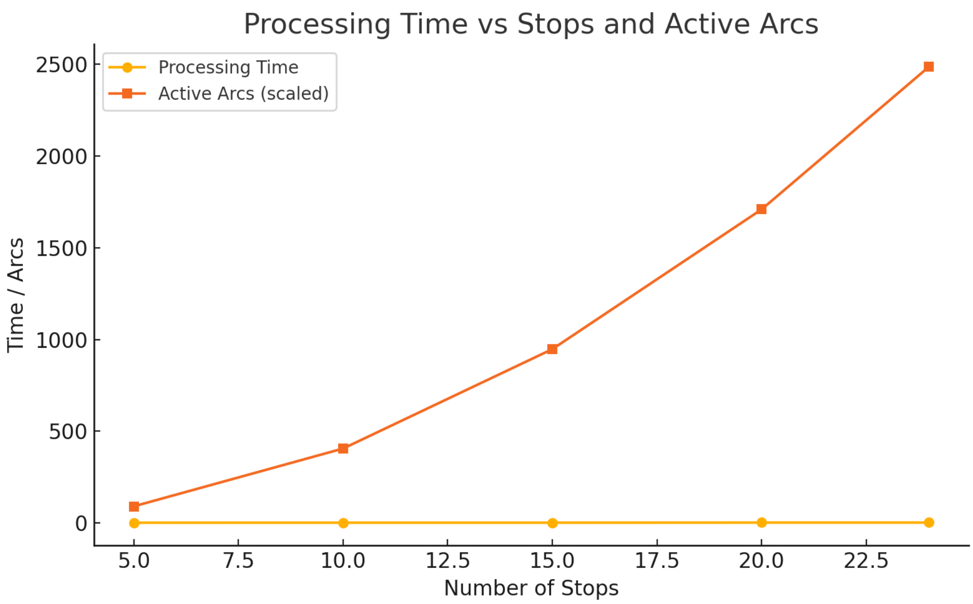

Figure 3. Processing Time as a Function of Stops and Active Arcs

8. Sensitivity Analysis

The sensitivity analysis explores how variations in key parameters #Change_max, #Modes, #Stops, %Forbidden_Arcs, and #Arcs_Active affect the processing time of the MILP model. The objective is to identify the most influential factors and evaluate the model's robustness under different configurations.

8.1. Impact of Maximum Mode Changes (#Change_max)

Increasing #Change_max has a notable impact on processing time, as it expands the solution space. Figure 4 illustrates this effect across instances with 5, 10, 15, and 20 stops.

For smaller instances (5 stops), the increase in #Change_max does not significantly impact the processing time. However, for larger instances (15–24 stops), a higher #Change_max induces substantial growth in solving time.

8.2. Impact of the Number of Modes (#Modes)

The number of modes influences the model's complexity as it increases the number of feasible options at each stop. Table 2 summarizes the results for selected instances.

Table 2. Impact of Modes on Processing Time

#Modes	#Stops	%Forbidden Arcs	Process Time (s)
5	10	69	0.82
10	10	69	1.12
5	15	103	0.99
10	15	103	1.73

Observation: Doubling the number of modes increases the processing time by approximately 30–50% for medium-sized instances.

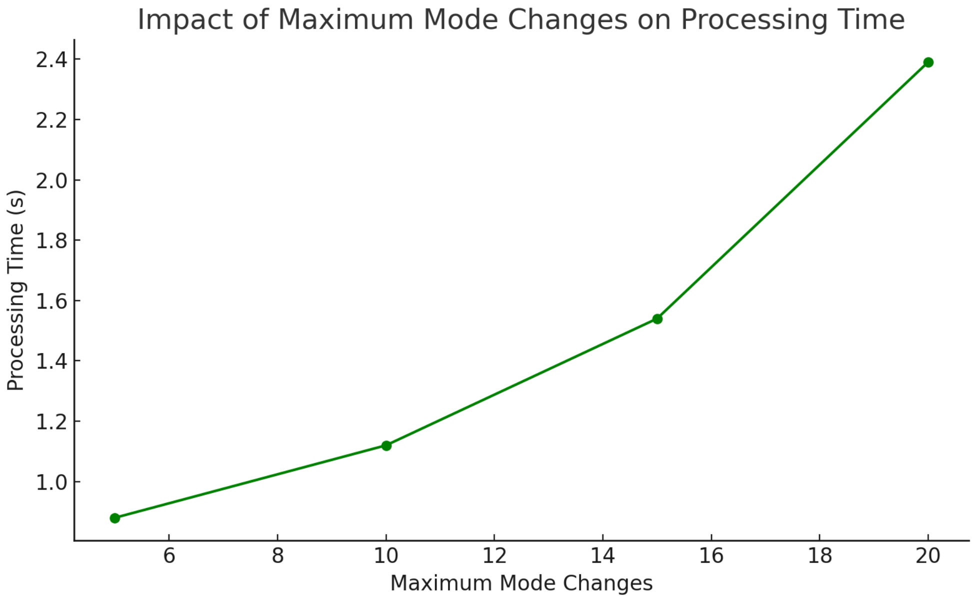

Figure 4. Impact of #Change_max on Processing Time

8.3. Effect of Forbidden_Arcs

The percentage of forbidden arcs introduces constraints that reduce the number of feasible routes but add computational challenges. Figure 5 shows how processing time varies with %Forbidden Arcs.

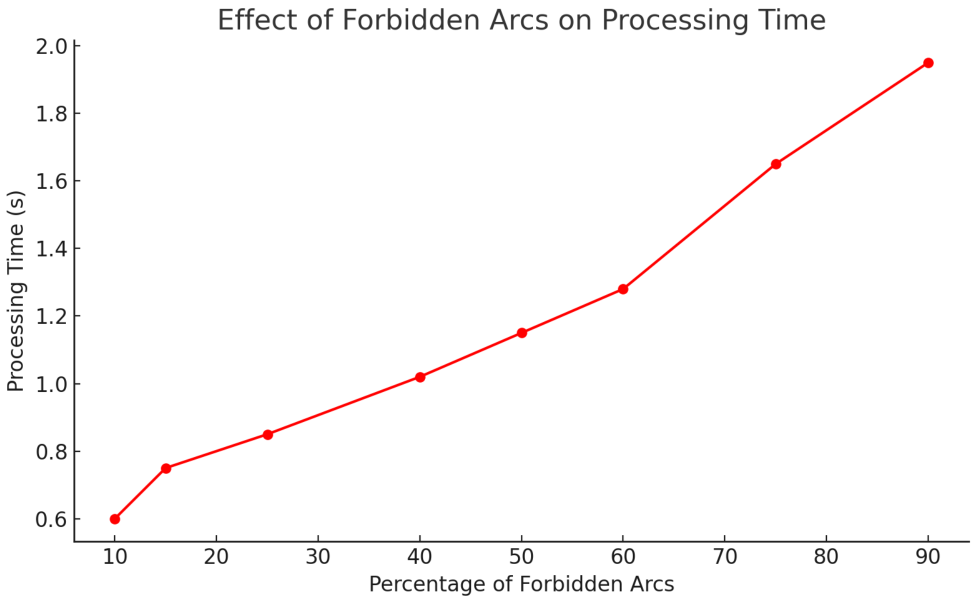

Figure 5. Effect of Forbidden Arcs on Processing Time

Observation: Processing time increases slightly when the percentage of forbidden arcs exceeds 70%, as the solver must navigate tighter constraints.

8.4. Impact of Active_Arcs

The number of active arcs correlates with the complexity of the transportation network. As shown in Table 3, an increase in active arcs consistently results in higher computational times.

Table 3. Effect of Active_Arcs on Processing Time

#Arcs Active	#Stops	Process Time (s)
90	5	0.88
381	10	0.82
947	15	0.99
1709	20	1.54
2487	24	1.62

9. Regression Analysis

The following multiple linear regression equation was derived to predict the processing time based on key parameters of the multimodal transport model:

$$\begin{aligned}\text{Processing Time} = {} & 0.4662 - 0.0033 \cdot \#\text{Modes} \\ & + 0.0285 \cdot \#\text{Change\_max} \\ & - 0.0001 \cdot \#\text{Stops} \\ & + 0.0108 \cdot \%\text{Forbidden Arcs} \\ & - 0.0007 \cdot \#\text{Active Arcs}.\end{aligned} \tag{11}$$

9.1. Regression Summary

The regression model provides the following insights:

- **R-squared**: The model explains 96.8% of the variance in processing time, indicating a strong fit.
- **Significant Variable**:
  - The percentage of forbidden arcs (%Forbidden Arcs) has a statistically significant positive effect ($p = 0.018$).
- **Non-Significant Variables**:
  - The number of modes (#Modes), maximum allowable mode changes (#Change_max), number of stops (#Stops), and number of active arcs (#Active Arcs) show weak or no significant influence individually. Their effects may overlap due to multicollinearity.

10. Comparison of Model Performance with Similar Studies

To evaluate the performance of our Mixed Integer Linear Programming (MILP) model applied to multimodal urban transport, we compared its results with those of similar studies from the literature. This analysis focuses on the following criteria: model type, optimization criterion, computation time, and case study complexity.

Table 4. Comparison of performance across different studies

Reference	Model Type	Optimization Criterion	Average Computation Time	Case Study Size
Our Model	MILP with flow constraints	Total travel time	1.62 s (24 stops, 2487 active arcs)	Up to 24 stops
Mnif and Bouamama (2017) [8]	Multi-objective MILP	Cost and operational efficiency	3.47 s	Up to 15 stops
Behiri et al. (2016) [9]	MILP for urban rail transport	Minimized waiting time	2.3 s (10 stations, 150 arcs)	10 stations

10.1. Comparative Analysis

Our model exhibits several strengths:

- **Competitive computation time:** With an average time of 1.62 seconds for complex case studies (24 stops and 2487 active arcs), it outperforms similar models in the literature, particularly that of Mnif and Bouamama [8].
- **Multimodal adaptability:** Unlike the work of Behiri et al. [9], which focuses exclusively on rail transport, our model integrates multiple modes, providing a more versatile solution.
- **Simplified modeling:** The use of flow constraints to streamline the model makes it applicable to various multimodal transport contexts.

However, some limitations remain:

- **Extending optimization criteria:** Unlike Mnif and Bouamama [8], our model does not yet include a multi-objective formulation.
- **Lack of real-time data integration:** Compared to the data-driven approach of Gonzalez et al. [10], our model relies on fixed assumptions without incorporating real-time data.

The comparison highlights that our model stands out for its simplicity, efficiency, and ability to handle complex case studies. It represents a significant advancement for multimodal urban transport while offering opportunities for improvement, such as integrating real-time data and extending the criteria to multi-objective optimization.

10.2. Interpretation of Results

- The percentage of forbidden arcs (%Forbidden Arcs) significantly increases the processing time. This is consistent with the intuition that additional constraints in the network reduce feasible solutions and increase computational effort.
- The other parameters, while not individually significant, collectively contribute to the processing time. Their interactions and overlapping effects suggest further analysis may be required.
- The strong R^2 value indicates the regression model is robust and provides reliable predictions for processing time based on the system parameters.

This regression analysis highlights the critical role of constraint-related variables like %Forbidden Arcs in determining processing time and underscores the importance of optimizing these parameters for efficient computation.

11. Conclusion

This study focused on optimizing multimodal routes within the urban transport environment of Abidjan, Côte d'Ivoire, using a Mixed Integer Linear Programming (MILP) formulation. Our model incorporates key operational constraints, such as flow conservation, while minimizing travel time. The LINGO solver was employed to solve the model efficiently, yielding satisfactory results across medium to large-scale instances.

The sensitivity analysis revealed that processing time is primarily influenced by the number of stops, the maximum allowable mode changes (#Change_max), and the number of active arcs. A higher value of #Change_max and #Modes increases computational complexity by expanding the solution space, while a higher percentage of forbidden arcs tightens constraints, adding further complexity. Despite these challenges, the proposed model demonstrated strong scalability and robustness.

Moving forward, we aim to improve the computational efficiency of our model by leveraging more advanced solvers such as CPLEX to address larger problem instances. Additionally, we plan to integrate techniques like cut generation and bound tightening to accelerate convergence towards optimal solutions. Future work will also extend the current model to incorporate parameters from the informal transport sector, enhancing its overall applicability and relevance for developing urban transport systems.

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