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Optimization of Weibull Distribution Parameters with Application to Short-Term Risk Assessment and Strategic Investment Decision-Making

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Abstract Accurate parameter estimation is fundamental in financial modeling, especially in investment analysis, where the Modified Internal Rate of Return (MIRR) plays a key role in evaluating investment performance. This study aims to enhance risk and return predictions in Sharia-compliant property investments by exploring the efficacy of various optimization techniques for estimating Weibull distribution parameters within the MIRR framework. To achieve this, we employed a comparative analysis of optimization methods, including Simulated Annealing (SA), Differential Evolution (DE), Genetic Algorithm (GA), and traditional Numerical Methods (NM). Performance was assessed through metrics such as Root Mean Squared Error (RMSE), Akaike Information Criterion (AIC), R-squared (R^2) values, and Kolmogorov-Smirnov (KS) statistics. The results reveal that metaheuristic algorithms (SA, DE, GA) significantly outperform traditional numerical methods in terms of parameter estimation accuracy. Specifically, SA achieved the lowest RMSE of 0.042, with a Weibull shape parameter estimate of 1.254 and variance of 0.004, followed closely by DE with an RMSE of 0.048, and GA with 0.046. In contrast, NM exhibited a higher RMSE of 0.067, with a shape parameter estimate of 1.310 and a variance of 0.006. The AIC values for metaheuristic methods ranged from 14.25 to 14.68, compared to 15.12 for NM, and R^2 values for metaheuristic methods ranged from 0.932 to 0.945, compared to 0.910 for NM. KS statistics further underscored the superior model fit of metaheuristics, with SA showing the lowest KS value of 0.045. The study underscores the critical role of metaheuristic optimization in improving the accuracy of parameter estimation based on MIRR models. This enhancement provides more reliable risk assessments and returns predictions, offering valuable insights for informed investment decision-making and contributing to optimized financial outcomes in the property sector.

Keywords Risk management; Return on Investment; Diversification of Risk; Investment; Risk and return; Parameter Estimation; Modified Internal Rate of Return; Weibull Distribution; Optimization Technique

AMS 2010 subject classifications: 62P05

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1. Introduction

Investment return analysis often relies on sophisticated statistical models to accurately assess the profitability and risk associated with various investment opportunities. One such model is the Weibull distribution, renowned for its versatility in modelling different types of data. This study focuses on optimizing Weibull distribution parameters using various metaheuristic algorithms to enhance risk and investment return analysis, specifically within the Malaysian property sector. By leveraging advanced optimization techniques, we aim to improve the precision of parameter estimates and, consequently, the accuracy of investment evaluations.

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The Weibull distribution, introduced by Waloddi Weibull [1], has proven valuable in numerous fields, including reliability engineering [2] and financial risk assessment [3], [4]. Its ability to model various types of data makes it suitable for analyzing investment returns. The distribution's flexibility in shape and scale parameters allows for adaptation to different data characteristics, which is crucial for accurately forecasting investment performance. In financial applications, the Weibull distribution model is applied to assess risk and predict returns by fitting historical data to the distribution and analyzing the resulting parameters. This approach has been explored in various studies to forecast financial risk and enhance portfolio management strategies [5]–[8].

Accurately estimating Weibull distribution parameters is crucial for reliable investment return analysis. Traditional methods, such as maximum likelihood estimation (MLE), may face limitations when dealing with complex datasets or optimizing over large parameter spaces. To overcome these challenges, metaheuristic optimization algorithms offer robust alternatives. Inspired by natural processes and problem-solving heuristics, these algorithms have shown promise in optimizing model parameters. For instance, Oliva et al. [9] reviewed several metaheuristic algorithms for parameter estimation, including evolutionary algorithms (GA, DE, SCW), physics-based algorithms (WDO, FPA, GSA), swarm-based algorithms (ABC, PSO, CSO, WOA), and human-based algorithms (HS). Alrashid [10] applied metaheuristic optimization algorithms to estimate statistical distribution parameters for characterizing wind speeds. Yonar [11] evaluated the performance of various metaheuristic methods in estimating gamma distribution parameters. Additionally, Freitas de Andrade [12] applied four heuristic optimization algorithms to determine Weibull curve parameters.

The Malaysian property sector presents a dynamic environment for investment analysis, characterized by diverse market conditions and evolving trends. The integration of Weibull distribution parameter optimization can provide valuable insights into investment return analysis within this sector. By applying optimized Weibull parameters, investors and analysts can achieve more accurate forecasts of investment performance and better understand risk profiles.

This study utilizes advanced optimization techniques to refine Weibull distribution parameters, aiming to enhance the accuracy of investment return analysis in the Malaysian property sector. By fitting investment return data to the Weibull distribution and optimizing the parameters with state-of-the-art metaheuristic algorithms, the study seeks to provide deeper insights into investment performance and risk management. Optimizing Weibull distribution parameters through these advanced metaheuristic algorithms marks a significant leap in investment return analysis. By improving the precision of parameter estimation, this approach not only enhances the understanding of investment behaviour but also supports more strategic decision-making. The application of these techniques to the Malaysian property sector highlights their potential for refining investment evaluations.

Our study aims to contribute to the advancement of more robust investment analysis methodologies by harnessing the power of metaheuristic optimization to enhance the accuracy and reliability of Weibull distribution parameter estimates. Although the Weibull distribution plays a critical role in modelling various datasets, selecting the most appropriate parameter estimation methods remains challenging in the financial analysis due to the inherent uncertainty of financial data. The complexity of financial datasets, particularly within the industrial sector, necessitates the use of robust techniques for accurately estimating Weibull distribution parameters. Therefore, investigating the effectiveness and reliability of metaheuristic algorithms—such as Genetic Algorithm (GA), Differential Evolution (DE), and Simulated Annealing (SA)—in optimizing these parameters as applied to financial data is essential.

Traditional metrics for investment analysis, such as Net Present Value (NPV) and Internal Rate of Return (IRR), while widely used, have limitations in fully capturing the dynamics of investment behaviour within the property sector. The Modified Internal Rate of Return (MIRR) offers a promising alternative, yet challenges persist in optimizing its application for long-term investment assessment.MIRR is considered a better metric for investment analysis because it uses more realistic parameters in the determination of the value of an investment than IRR or NPV. It also addresses the limitations of IRR by estimating reinvestment at cost of capital rate rather than at internal return rate[13]–[15]. A research gap exists in understanding how to best utilize MIRR within the context of the Malaysian Property Sector (MPS) and in assessing its effectiveness in evaluating investment return potential over extended time horizons.

Integrating metaheuristic algorithms for parameter estimation with MIRR data represents an innovative approach to evaluating investment returns in the industrial sector. However, the effectiveness and accuracy of this method, particularly within the context of Sharia-compliant companies, have not been thoroughly investigated. This presents a valuable research opportunity to explore how metaheuristic algorithms can be employed to estimate Weibull distribution parameters using MIRR data from Sharia-compliant companies and to evaluate the short-term investment potential and associated risks within this sector.

The contributions of this study are as follows:

1. To evaluate the effectiveness of metaheuristic algorithms in estimating Weibull distribution parameters using financial data from the property sector over short-term investment periods.
2. To assess the applicability and reliability of the Modified Internal Rate of Return (MIRR) by incorporating stock splits, bonus issues, and treasury shares in evaluating short-term investment attractiveness in Sharia-compliant Malaysian property development sector (MPS) companies.
3. To compare the performance of metaheuristic algorithms in estimating Weibull distribution parameters with traditional numerical methods, focusing on accuracy and efficiency in capturing investment return behavior and risk profiles within the Malaysian property sector.

The structure of this paper is organized as follows: Section 2 presents the Materials and Methods, detailing the MIRR data collection procedure, the methodology behind data collection, and the derivation of share accumulation dynamics relevant to investment modelling and the Weibull distribution. In Section 3, we discuss the implementation of optimization algorithms, including Simulated Annealing, Differential Evolution, and Genetic Algorithm. Section 4 presents the Goodness of the Fit Test. Section 5 presents the Results and Discussion, featuring the application of MIRR data from the MPS, including the assessment of mean and variance for risk evaluation and the interpretation of results within the framework of investment analysis. Section 6 concludes with a summary of key findings, highlighting the contributions of this study and its implications for investment analysis, along with outlining future research directions.

2. Materials and Methods

This section outlines the methodology for constructing the Modified Internal Rate of Return (MIRR) model, incorporating data aggregation, the Weibull distribution framework, and metaheuristic optimization techniques for parameter estimation within the Weibull distribution paradigm.

2.1. Modified Internal Rate of Return Data Collection Method

This study explores the performance of the Malaysian property sector (MPS), with a specific focus on Shariah-compliant companies listed on Bursa Malaysia between 2010 and 2015. The MPS was selected due to its pivotal role in the Malaysian economy and its dynamic, volatile nature, marked by fluctuating market conditions and ongoing regulatory changes [16], [17]. The sector's growth, coupled with the increasing interest in Shariah-compliant investments, underscores the importance of evaluating its financial performance and investment potential [18]. By analyzing this sector over six years, the study provides a perspective across different economic cycles, offering valuable insights into investment strategies, particularly through applying the Modified Internal Rate of Return (MIRR).

The research focuses on 62 Shariah-compliant companies listed on Bursa Malaysia, with a thorough examination of their annual reports and dividend declarations from 2010 to 2015. The data collection process involved several key steps:

- **Stock Prices and Dividends**: Historical stock prices and dividend declarations were obtained from the Bursa Malaysia website and the Wall Street Journal, covering the period from January 1, 2010, to December 30, 2015.
- **Shariah Compliance Verification**: The Shariah-compliant status of the selected companies was verified using official lists retrieved from the Bursa Malaysia website.

- **Financial Adjustments**: Careful adjustments for stock splits, bonus issues, and treasury share transactions were incorporated to ensure an optimal representation of share values and dividend distributions.

This approach allows for a robust analysis of the financial performance of Sharia-compliant companies within the Malaysian property sector, providing critical insights for investors and policymakers.

2.2. *Mathematical formulation of Investment Cashflow Model*

Mathematical models were developed to represent share accumulation, considering stock splits, bonus issues, and the impact of treasury shares. The methodologies used for these calculations are based on the approach outlined in [19]. The models consider the effects of stock splits and bonus issues, as well as the influence of treasury shares on the total number of outstanding shares. While dividends are generally paid in cash, some companies issue mandatory treasury share dividends. In such cases, cash dividends are distributed, but the number of share units is reduced by a specified percentage, as detailed in the company's annual reports.

2.3. *Stock Investment Cashflows Model*

Let $k = 1, 2, \ldots, K$ denote the discrete yearly time series, where K represents the total number of investment years. Define u_{1k} and u_{2k} as the specific dates within the k-th year, with u_{1k} indicating the date of the initial purchase of share units and u_{2k} representing the date of the annual financial report publication. The end of the investment period within the k-th year is denoted by u_K^*.

At the beginning of the k-th year, an amount E_k is allocated for acquiring C_k share units. Bursa Malaysia mandates that equities be traded in board lots, with one lot consisting of 100 share units[20]. Investors must therefore adhere to these multiples for transactions. Assuming no brokerage costs, the number of share units purchased C_k and the actual capital invested E_k are computed using:

$$C_k = \left\lfloor \frac{E_k - B_{k-1} - \delta_{k-1} - A_{k-1}}{P_{u_{1k}}} \right\rfloor \times 100 \tag{1}$$

$$E_k = C_k \times P_{u_{1k}} \tag{2}$$

where $P_{u_{1k}}$ denotes the share price on the date u_{1k}, B_{k-1} is the remaining balance from the previous year, δ_{k-1} is the dividend declared in the previous year, and A_{k-1} is an adjustment term for any residual amounts from previous investments.

The nominal value of the non-invested balance B_k is updated as:

$$B_k = B_{k-1} + \delta_{k-1} - E_k + A_{k-1} \tag{3}$$

For $k = 1$, the initial number of share units and capital invested are calculated by:

$$C_{1k} = \left\lfloor \frac{E_{1k}}{P_{u_{1k}}} \right\rfloor \times 100 \tag{4}$$

$$E_{1k} = C_{1k} \times P_{u_{1k}} \tag{5}$$

The first-year cash balance B_1 is:

$$B_1 = C - E_{1k} \tag{6}$$

In the case of dividends, it is assumed that dividends are distributed on the financial report publication date u_{2k}. Let d_k be the dividend per share. The total dividend D_k is split into two components: $D_k^{(1)}$, the dividend from existing share units, and $D_k^{(2)}$, the dividend from newly acquired share units. The dividend from existing share units $D_k^{(1)}$ is:

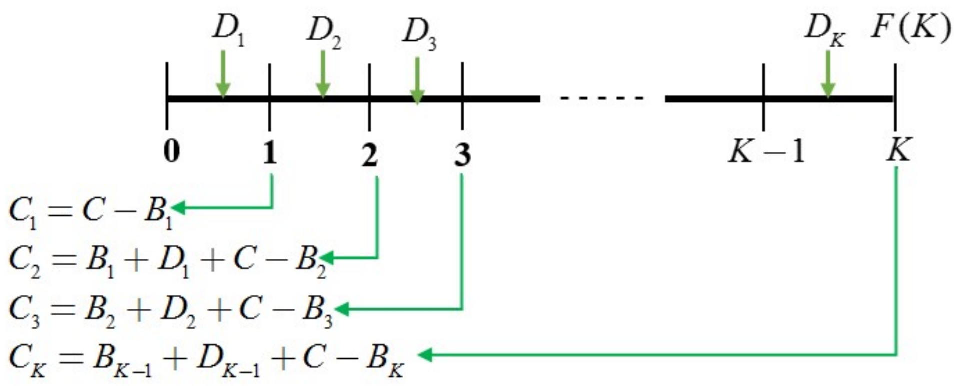

Figure 1. Stock Investment cash flows diagram over K years

$$D_k^{(1)} = S_{k-1} \times d_k \tag{7}$$

where S_{k-1} is the total number of shares held at the beginning of the k-th year.

For newly acquired share units, if the purchase date u_{1k} and the financial report date u_{2k} are within less than one year, a prorated dividend $D_k^{(2)}$ is computed. The linear approximation of this prorated dividend is:

$$D_k^{(2)} = \frac{C_k \times d_k \times (u_{2k} - u_{1k})}{\text{Days\_in\_Year}} \tag{8}$$

where Days_in_Year is the number of days in the financial year (typically 365).

Thus, the total dividend D_k is:

$$D_k = D_k^{(1)} + D_k^{(2)} = S_{k-1} \times d_k + \frac{C_k \times d_k \times (u_{2k} - u_{1k})}{\text{Days\_in\_Year}} \tag{9}$$

To incorporate additional complexity, consider the following adjustments:

Adjust the dividends and investments for inflation i_k in each year:

$$D_k^{\text{adj}} = \frac{D_k}{(1 + i_k)^{k-1}} \tag{10}$$

Incorporate a compounding effect in the calculation of the investment amount:

$$E_k^{\text{comp}} = E_{k-1} \times (1 + r_k) + C_k \times P_{u_{1k}} \tag{11}$$

where r_k is the annual return rate on investment.

Account for capital gains tax t_k on the total gains G_k:

$$G_k = (C_k \times P_{u_{2k}} - E_k) \tag{12}$$

$$T_k = t_k \times G_k \tag{13}$$

$$D_k^{\text{net}} = D_k - T_k \tag{14}$$

Figure 1 illustrates the investment cash flows over K years, depicting how contributions, dividends, and adjustments are employed to purchase additional share units. The investor initially allocates C_1 to acquire E_1 share units, leaving a balance B_1 of non-invested capital (i.e., $C - C_1$). In the first year, the investor receives a dividend D_1^{adj} which, combined with B_1 and adjusted for inflation, is reinvested to purchase additional share units E_2 at the beginning of the subsequent year. This process continues cyclically, with adjustments for inflation, compounding effects, and capital gains taxes applied throughout the investment period. The final dividend D_K^{net} is distributed after the last payment and remains uninvested.

2.4. Derivation for Share Accumulation Dynamics

Share accumulation involves tracking the evolution of share units over time due to various issuance events. This section outlines the dynamics of share units before and after each issuance event within a given year[21], [22]. Let $S_k^{(1)}$ represent the share units before the issuance event in year k, and $S_k^{(2)}$ represent the units following the event. The relationship between these units is governed by the share issuance function g_k, which is defined as follows:

$$S_k^{(2)} = g_k(S_k^{(1)}) \tag{15}$$

To derive g_k, we consider scenarios such as share splits, consolidations, bonus issues, and dividends.

- **Share Split**: Increases the number of shares and is represented by a factor $\text{Spl}_k = \frac{N_k}{M_k}$, where N_k and M_k denote the number of shares after and before the split, respectively.
- **Share Consolidation**: Decreases the number of shares, represented by $\text{Con}_k = \frac{M_k}{N_k}$, where $\text{Con}_k = 1 - \text{Spl}_k$.
- **Bonus Issue**: Affects the share units by the bonus ratio Bns_k, increasing the number of shares held by the shareholders.
- **Dividends**: Companies may distribute dividends, which can be used to reinvest in additional shares. These dividends are not treasury share dividends but can originate from various sources, as detailed below.

Dividends can be received from several different types of investments. This include: dividends from Investments in Other Companies. In this situation, company may hold equity in other companies, and generating dividends from these investments can be modelled as follows;

$$\text{Div}_k^{\text{other}} = \text{Investment}_k^{\text{other}} \times \text{Dividend Yield}_k^{\text{other}} \tag{16}$$

Where:

- $\text{Investment}_k^{\text{other}}$ is the total value of investments in other companies at year k.
- $\text{Dividend Yield}_k^{\text{other}}$ is the dividend yield of these investments.

The company may receive dividends from its equity holdings in subsidiaries which are modelled as follows;

$$\text{Div}_k^{\text{subs}} = \text{Investment}_k^{\text{subs}} \times \text{Dividend Yield}_k^{\text{subs}} \tag{17}$$

Where:

- $\text{Investment}_k^{\text{subs}}$ is the total value of investments in subsidiaries at year k.
- $\text{Dividend Yield}_k^{\text{subs}}$ is the dividend yield from these subsidiaries.

If the company is part of a group structure, one company within the group may pay dividends to another modelling as follows;

$$\text{Div}_k^{\text{group}} = \sum_{i=1}^{n} \left(\text{Investment}_k^{\text{group},i} \times \text{Dividend Yield}_k^{\text{group},i} \right) \tag{18}$$

Where:

- $\text{Investment}_k^{\text{group},i}$ is the investment in the i-th company within the group at year k.
- $\text{Dividend Yield}_k^{\text{group},i}$ is the dividend yield from the i-th company.

2.5. Share Issuance Function

Considering the above sources of dividends, the share issuance function g_k for year k, accounting for share splits, consolidations, bonus issues, and reinvested dividends, is given by:

$$g_k(S) = S \times (1 + \text{Spl}_k - \text{Con}_k) \times (1 + \text{Bns}_k) \times \left(1 + \frac{\text{Div}_k^{\text{total}}}{S \times P_k}\right) \tag{19}$$

Where:

- $\text{Div}_k^{\text{total}} = \text{Div}_k^{\text{other}} + \text{Div}_k^{\text{subs}} + \text{Div}_k^{\text{group}}$ is the total dividend received from all sources.
- P_k is the share price at year k.

The cumulative share issuance function over k years can be expressed as:

$$g_K(S) = \prod_{i=1}^{k} \left[S \times (1 + \text{Spl}_i - \text{Con}_i) \times (1 + \text{Bns}_i) \times \left(1 + \frac{\text{Div}_i^{\text{total}}}{S \times P_i}\right) \right] \tag{20}$$

The accumulated share units at the end of year k are given by:

$$S_k^{(2)} = g_k(S_k^{(1)}) \tag{21}$$

At the beginning of the first year, the shareholder purchases E_1 share units:

$$S_1^{(1)} = E_1 \tag{22}$$

The share accumulation for the first year, considering any issuance, is:

$$S_1^{(2)} = g_1(E_1) \tag{23}$$

At the start of the second year, the shareholder purchases an additional E_2 share units:

$$S_2^{(1)} = S_1^{(2)} + E_2 \tag{24}$$

The share accumulation at the end of the second year is:

$$S_2^{(2)} = g_2(S_2^1) = g_2(S_1^{(2)} + E_2) \tag{25}$$

For subsequent years, the recursive relationships are:

$$S_{k+1}^{(1)} = S_k^{(2)} + E_{k+1} \tag{26}$$

$$S_{k+1}^{(2)} = g_{k+1}(S_{k+1}^{(1)}) \tag{27}$$

2.6. Terminal Fund and Net Present Value (NPV)

The terminal fund F_K is calculated as:

$$F_K = S_K^{(2)} \times P_{u_K^*} + B_K + D_K \tag{28}$$

Where:

- $P_{u_K^*}$ is the share price on the date u_K^*,
- B_K is the non-invested balance,
- D_K is the final dividend payout.

The Net Present Value (NPV) of the investment is:

$$NPV = \sum_{k=1}^{K} \frac{S_k^{(2)} \times P_{u_k} + B_k + D_k - C_k}{(1 + r_k)^{k-1}} \tag{29}$$

Where:

- r_k is the discount rate for year k,
- C_k represents the capital invested in year k.

2.6.1. Computation of Share Accumulation for Tropicana Corporation Berhad Tropicana Corporation Berhad (Trop), previously known as Dijaya Corporation Berhad (Dijacor) with the stock symbol 5401, had its Modified Internal Rate of Return (MIRR) calculated for the period from 2010 to 2015. Trop paid dividends annually, including dividends received from investments in other companies and from equity holdings in subsidiaries. The following tables provide the necessary details to calculate the investment cash inflows and outflows.

Table 1. Dividend, Share Price, and Share Issuance Indicators for Tropicana Corporation Berhad

Year	Start Date	End Date	Initial Price	Final Price	Dividend Rate	Treasury Share	Adjusted Shares
1	03/01/10	31/12/10	1.01448	1.34614	0.0375	0	1
2	03/01/11	31/12/11	1.34614	1.00473	0.0225	0	1
3	03/01/12	31/12/12	1.00473	1.21933	0.08175	0	1
4	03/01/13	31/12/13	1.21933	1.19982	0.0400	0	1
5	03/01/14	31/12/14	1.19982	1.04374	0.0630	0.013	0.987
6	03/01/15	31/12/15	1.04374	0.98814	0.0450	0	1

Based on the data in Table 2, RM10,000 was invested at the beginning of the investment period (03/01/2010) and withdrawn on 31/12/2015.

Table 2. Cash Flow of Investing in Tropicana Corporation Berhad (2010-2015)

Year	Initial Cash	Invested Amount	Div. Received from Other Investments	Cumulative Invested	Adjusted Cumulative Invested	Div. Payout
1	9,800.00	9,800.00	58.10	9,800.00	9,800.00	364.48
2	10,365.28	10,365.28	57.30	20,165.28	20,165.28	392.80
3	11,000.00	11,000.00	2.91	31,165.28	31,165.28	2,281.94
4	12,238.16	12,238.16	46.69	43,403.44	43,403.44	1,524.65
5	13,142.26	13,142.26	90.20	56,545.70	55,194.80	3,093.90
6	13,500.00	13,500.00	41.84	69,045.70	69,045.70	2,780.44

The terminal investment $F_K^{(t)}$ and the Modified Internal Rate of Return (MIRR) r_K for an investment period K are calculated using the following formulas:

$$F_K^{(t)} = S_K^{(2)} \cdot P_{K+1,K} + B_K + D_K \tag{30}$$

where:

- $S_K^{(2)}$ is the adjusted cumulative investment value at the end of year K.
- $P_{K+1,K}$ is the final price to initial price ratio for the period after K years.

- B_K represents the dividend payout received during the investment period.
- D_K denotes the dividends received from other investment.

For example, for $K = 4$:

$$F^{(4)} = 43,403.44 \times 1.19982 + 46.69 + 1,524.65 \approx 53,694.74 \tag{31}$$

2.7. Modified Internal Rate of Return Computation

The MIRR is found by solving:

$$F(K) = C_0 \left(1 + r_K\right)^{-K} \tag{32}$$

where:

- $F(K)$ is the terminal investment value.
- C_0 is the initial investment amount (RM10,000).
- r_K is the reinvestment rate.

The MIRR formula can be rearranged as:

$$53,694.74 = 10,000 \left(1 + \frac{1}{(1+r_K)} + \frac{1}{(1+r_K)^2} + \frac{1}{(1+r_K)^3}\right) \tag{33}$$

Table 3. Terminal Investment and MIRR from 2010 to 2015

Investment Period (Years)	Terminal Investment (RM)	MIRR
1	10,363.28	0.3560
2	20,365.28	-0.0783
3	33,447.19	0.0806
4	53,694.74	0.0156
5	68,668.58	0.0074
6	69,949.02	0.0134

The Modified Internal Rate of Return (MIRR) is an important metric for evaluating the performance of short-term investments. This analysis demonstrates its application using data from Tropicana Corporation Berhad (Trop) for the period from 2010 to 2015. The analysis begins with an initial investment and tracks annual cash flows, including dividends and adjustments related to shares.

Adjustments for dividend payouts and treasury shares are crucial for accurately reflecting their impact on investment returns. The terminal investment value is calculated based on the final share price, adjusted cumulative investments, and dividend payouts. This calculation is key to assessing overall investment performance. The MIRR is computed by solving an equation that aligns the terminal investment value with the initial investment over the investment period. This provides a comprehensive measure of returns. To expand the analysis, MIRR data from 62 property companies will be examined using the Weibull distribution. Assuming a uniform MIRR across all companies and identical starting times for investments, the MIRR can be modelled as a random variable for each stock at each investment year over the specified period. This random variable is denoted as R_{tiK}, with the initial investment amount represented by R.

The metaheuristic algorithms will be used to optimize the parameters of this distribution, improving the accuracy of modelling and forecasting investment returns. By integrating the Modified internal rate of return framework with the Weibull distribution, the analysis aims to provide better insights into investment performance and risk, offering valuable guidance for refining short-term investment strategies within the sector.

The expected return $E[R_{tiK}]$ is defined as:

$$E[R_{tiK}] = E\left[\frac{F_i(t)}{R}\right] \tag{34}$$

where:

- $F_i(t)$ is the terminal investment value for the i-th stock after t years.
- R is the initial investment amount.

Considering the terminal investment value $F_i(t)$ as a function of MIRR and dividends, the expected return over T periods can be computed as:

$$E[R_{tiK}] = \frac{1}{T}\sum_{t=1}^{T} E\left[\frac{F_i(t)}{R}\right] \tag{35}$$

The terminal investment value $F_i(t)$ can be expressed as:

$$F_i(t) = P_i(t)\left(1 + \sum_{j=1}^{t}\frac{D_{ij}}{R}\right) + \sum_{k=1}^{t} T_{ik} \tag{36}$$

where:

- $P_i(t)$ is the price of the stock at time t.
- D_{ij} is the dividend paid at time j.
- T_{ik} represents any treasury share dividends at time k.

Substituting this into the expected return formula, we get:

$$E[R_{tiK}] = \frac{1}{T}\sum_{t=1}^{T} E\left[\frac{P_i(t)\left(1 + \sum_{j=1}^{t}\frac{D_{ij}}{R}\right) + \sum_{k=1}^{t} T_{ik}}{R}\right] \tag{37}$$

The aggregate MIRR for the dataset can be determined by averaging the individual MIRRs across all periods. The MIRR for each period K is given by:

$$r_K = \left(\frac{F_K}{R}\right)^{\frac{1}{K}} - 1 \tag{38}$$

where F_K is the terminal investment value at period K.

The aggregated MIRR across all periods is:

$$\text{Aggregate MIRR} = \frac{1}{T}\sum_{K=1}^{T}\left(\left(\frac{F_K}{R}\right)^{\frac{1}{K}} - 1\right) \tag{39}$$

The aggregate MIRR for the dataset is determined by averaging the individual MIRRs across all periods. It is essential to adjust the final results by including treasury shares and dividends to ensure accuracy in financial projections. This approach provides a comprehensive analysis of investment performance, accounting for both market price fluctuations and dividend distributions.

2.8. Weibull Distribution

Let $X = \{x_1, x_2, \dots, x_N\}$ denote a random sample of size N drawn from the transformed MIRR data. The cumulative distribution function (CDF) of the three-parameter Weibull distribution is given by:

$$F(x; k, \lambda, \alpha) = 1 - e^{-\left(\frac{x-\alpha}{\lambda}\right)^k} \tag{40}$$

The corresponding probability density function (PDF) is:

$$f(x;k,\lambda,\alpha) = \frac{k}{\lambda}\left(\frac{x-\alpha}{\lambda}\right)^{k-1} e^{-\left(\frac{x-\alpha}{\lambda}\right)^k} \tag{41}$$

Here, x represents the transformed MIRR data over the investment period t; k is the shape parameter that dictates the distribution's shape; λ is the scale parameter illustrating the distribution of MIRR data; and α is the location parameter. The Weibull PDF has the following properties:

1. If $k > 1$, the Weibull distribution function decreases with x.
2. If $k < 1$, the Weibull distribution function continues to decrease with x.
3. For $k = 1$, the Weibull distribution initially increases and then decreases, peaking at the mode α.
4. For all x, $f(x;k,\lambda,\alpha) > 0$.

When $k = 1$, the Weibull distribution simplifies to:

$$f(x;\lambda,\alpha) = \frac{1}{\lambda}e^{-\left(\frac{x-\alpha}{\lambda}\right)} \tag{42}$$

with the corresponding CDF:

$$F(x;\lambda,\alpha) = 1 - e^{-\left(\frac{x-\alpha}{\lambda}\right)} \tag{43}$$

If $k = 1$ and α is constant, the Weibull PDF further simplifies to a one-parameter Weibull distribution:

$$f(x;\lambda) = \frac{1}{\lambda}e^{-\left(\frac{x}{\lambda}\right)} \tag{44}$$

where λ is the scale parameter. This study focuses on the two-parameter Weibull distribution. The expected value of the Weibull distribution is:

$$E[X] = \alpha + \lambda\Gamma\left(1 + \frac{1}{k}\right) \tag{45}$$

where Γ denotes the gamma function.

The variance of the Weibull distribution is:

$$\text{Var}(X) = \lambda^2\left[\Gamma\left(1+\frac{2}{k}\right) - \left(\Gamma\left(1+\frac{1}{k}\right)\right)^2\right] \tag{46}$$

The Weibull distribution can model various types of data and transitions seamlessly to other distributions[23]: it becomes the standard exponential distribution when $k = 1$ and the Rayleigh distribution when $k = 2$.

The Likelihood and Log-Likelihood Functions of the Weibill distribution can be computed as follows.

Given a sample $X = \{x_1, x_2, \ldots, x_N\}$, the likelihood function for the Weibull distribution is:

$$L(\theta;X) = \prod_{i=1}^{N} \frac{k}{\lambda}\left(\frac{x_i-\alpha}{\lambda}\right)^{k-1} e^{-\left(\frac{x_i-\alpha}{\lambda}\right)^k} \tag{47}$$

where $\theta = \{k, \lambda, \alpha\}$ are the parameters of the Weibull distribution.

The log-likelihood function is obtained by taking the natural logarithm of the likelihood function:

$$\log L(\theta;X) = \sum_{i=1}^{N}\left[\log\left(\frac{k}{\lambda}\right) + (k-1)\log\left(\frac{x_i-\alpha}{\lambda}\right) - \left(\frac{x_i-\alpha}{\lambda}\right)^k\right] \tag{48}$$

Here, $\log L(\theta;X)$ is the sum of the log of the PDF evaluated at each observed value x_i.

Applications of the Weibull distribution in reliability Engineering are in Modeling time-to-failure data[2], in modelling of wind speed and precipitation[24], In Life Data Analysis used in studying lifetimes of mechanical systems and biological organisms[25] and in financial and actuarial applications[6] use in forecasting stock prices and investment behaviour. The next section presents parameters estimation methods[26], [27].

2.9. Paremeters Optimization Algorithm

Estimating the parameters of a Weibull distribution can be approached using various techniques, which are generally divided into two main categories: analytical/numerical methods and metaheuristic approaches. Analytical and numerical methods, such as Maximum Likelihood Estimation (MLE) and the Method of Moments, involve mathematical formulations and iterative procedures to estimate parameters. In contrast, metaheuristic optimization approaches utilize stochastic techniques inspired by natural processes to find optimal solutions in complex, nonlinear, and multi-dimensional spaces. This section discusses three metaheuristic methods for parameter optimization of the Weibull distribution: the Simulated Annealing (SA) algorithm, Differential Evolution (DE), and the Genetic Algorithm (GA).

2.9.1. Simulated Annealing Algorithm (SA) The Simulated Annealing Algorithm (SA) is a versatile metaheuristic inspired by the annealing process in metallurgy, where materials are gradually cooled to form a stable crystalline structure [28]. This algorithm adeptly navigates complex optimization landscapes to find global optima by emulating this cooling process. Its key strengths include simplicity in implementation and the ability to escape local optima, making it highly effective for exploring diverse solution spaces [29], [30]. In this study, SA is used to estimate the parameters of the Weibull distribution. The objective function is formulated as a minimization problem based on the following steps:

$$\min_{\mathbf{x}\in X} f(\mathbf{x}) - f_{\text{opt}} \tag{49}$$

or in maximization form:

$$\max_{\mathbf{x}\in X} f(\mathbf{x}) - f_{\text{opt}} \tag{50}$$

where the variable $\mathbf{x} \in X$ is to be estimated via the SA approach. The basic steps of the SA algorithm are as follows:

Step 1: Population Initialization

Randomly assign values to each parameter within specified ranges:

$$\mathbf{x}_i = (x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in}) \tag{51}$$

Start with a random initial placement $\mathbf{x}_i$. Initialize the optimization process at a very high temperature T_0 and a feasible trial point $\mathbf{x}(0)$:

$$T_k = T_{\text{min}} + (T_{\text{max}} - T_{\text{min}}) \cdot r \tag{52}$$

where T_{min} and T_{max} represent the initial and final temperatures, respectively. N is the number of temperatures, $\mathbf{x}_t \in [1, N]$, selected based on a problem-dependent cooling schedule.

Step 2: Generate New Point

Create a new point $\mathbf{x}_k$ at random in the vicinity of the present point. If the point is not feasible, produce a new random point until a feasible point is found.

Step 3: Fitness Computation

Calculate the fitness (objective) function using the formulated objective function and the difference:

$$\Delta f = f(\mathbf{x}_k) - f(\mathbf{x}(0)) \tag{53}$$

Step 4: Select the New Best Point

If $\Delta f < 0$, set $\mathbf{x}_k$ as the new optimal point $\mathbf{x}(0)$, set $f(\mathbf{x}(0)) = f(\mathbf{x}_k)$, and proceed to Step 5. Otherwise, compute the probability density function:

$$p(\Delta f) = \exp\left(-\frac{\Delta f}{T}\right) \tag{54}$$

Using the computed probability, generate a random number r. If $r < p(\Delta f)$, select $\mathbf{x}_k$ as the new best point $\mathbf{x}(0)$ and proceed to Step 5. If not, return to Step 2:

$$\mathbf{x}(k) = \begin{cases} \mathbf{x}_k & \text{if } r < p(\Delta f) \\ \mathbf{x}(0) & \text{otherwise} \end{cases} \tag{55}$$

Step 5: Stopping Criteria

If $i < L$, set $i = i + 1$ and return to Step 2. If $i > L$ or any stopping criterion is met, stop the process. Otherwise, move to Step 6.

Step 6: Update Temperature

Set $i = i + 1$ and update the temperature:

$$T_i = rT_{i-1} \tag{56}$$

Then, return to Step 2.

The final result of Weibull-SA contains the estimated parameters of the Weibull distribution.

2.9.2. Differential Evolution Algorithm (DE) Differential Evolution (DE), introduced by Storn and Price in 1997 [31], is a versatile optimization algorithm adept at solving complex, nonlinear problems. DE works by iteratively refining a population of candidate solutions through mutation, crossover, and selection processes to converge on optimal solutions. Its application in estimating Weibull distribution parameters is particularly noteworthy, as DE effectively navigates the solution space, balancing exploration and exploitation to achieve high accuracy and efficiency in parameter estimation, even in challenging optimization landscapes..

Step 1. Population Initialization

Define the search space for the shape and scale parameters. Each individual in the population is represented as:

$$x_i = (x_{i1}, x_{i2}, \dots, x_{in}) \tag{57}$$

where x_{ij} represents the j-th parameter of the i-th individual. The lower (x_i^l) and upper (x_i^u) bounds for each parameter are:

$$x_i^l = \left(x_{i1}^l, x_{i2}^l, \dots, x_{in}^l\right) \tag{58}$$

$$x_i^u = (x_{i1}^u, x_{i2}^u, \dots, x_{in}^u) \tag{59}$$

Step 2. Random Selection

Select initial parameter values uniformly within the defined intervals:

$$x_{i0} = x_i^l + \text{rand}(0,1) \cdot (x_i^u - x_i^l) \tag{60}$$

where $\text{rand}(0,1)$ is a random number between 0 and 1.

Step 3. Initial Population Evaluation

Evaluate the initial population using the objective function $f_{\text{Weibull-DE}}(x_i)$, where the log-likelihood function $L(x_i)$ for Weibull distribution is:

$$f_{\text{Weibull-DE}}(x_i) = \sum_{k=1}^{N} \left[\frac{\ln(\lambda) - \ln(\theta) + (\lambda - 1)\ln(x_{ik}) - \left(\frac{x_{ik}}{\theta}\right)^{\lambda}}{\theta^{\lambda}} \right] \tag{61}$$

where λ is the shape parameter, θ is the scale parameter, and x_{ik} are the observed data points.

Step 4. Mutation Phase

Generate a mutant vector by combining differences of randomly selected individuals:

$$v_i = x_{r1} + F \cdot (x_{r2} - x_{r3}) \tag{62}$$

where x_{r1}, x_{r2}, x_{r3} are randomly selected vectors from the population, and $F \in [0, 2]$ is the mutation factor. The mutation output v_i is then clamped within bounds:

$$v_{ij} = \text{clamp}(v_{ij}, x_{ij}^l, x_{ij}^u) \tag{63}$$

where $\text{clamp}(v, a, b)$ limits v to the interval $[a, b]$.

Step 5. Recombination Phase

Create trial vectors u_i by combining the current vector x_i with the mutant vector v_i:

$$u_{ij} = \begin{cases} v_{ij} & \text{if rand}(0,1) \leq C_r \text{ or } j = j_{rand} \\ x_{ij} & \text{otherwise} \end{cases} \tag{64}$$

where $C_r \in [0, 1]$ is the crossover probability, and j_{rand} ensures at least one parameter is taken from v_i.

Step 6. Selection Phase

Select between the trial vector u_i and the current vector x_i based on fitness:

$$x_i = \begin{cases} u_i & \text{if } f_{\text{Weibull-DE}}(u_i) \leq f_{\text{Weibull-DE}}(x_i) \\ x_i & \text{otherwise} \end{cases} \tag{65}$$

Step 7. Termination

The algorithm iterates up to a maximum of 10,000 generations or until a stopping criterion is met. The final result Weibul-DE is the set of parameters that minimizes the objective function.

2.9.3. Genetic Algorithm (GA) The Genetic Algorithm (GA) is a widely recognized metaheuristic algorithm introduced by John Holland in 1975[32]. It serves as a framework for solving various optimization and decision problems, drawing inspiration from the principles of biological evolution. GA utilizes a population of candidate solutions, represented as chromosomes, which evolve over iterations to optimize a given objective function.

In this work, GA has been used to optimize the parameters of the Weibull distribution by maximizing the likelihood function. The implementation, referred to as Weibull-GA, follows the steps outlined below to estimate the distribution parameters. The objective function for Weibull-GA is defined to guide the optimization process.

Step 1. Population Initialization

$$x_i = (x_{i1}, x_{i2}, \ldots, x_{iN}) \tag{66}$$

The GA population consists of chromosomes as presented in the equation, each containing random parameters. The aim is to optimize the likelihood function as follows:

$$L(x_i) = \sum_{j=1}^{n} \begin{cases} f_{\text{Weibull-GA}}(x_i) & \text{if } x_i \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{67}$$

Step 2. Fitness Computation

Each chromosome is evaluated according to the fitness function to ascertain its likelihood. The fitness of each chromosome is quantified as follows:

$$\text{fit}_i = \frac{1}{1 + f_{\text{Weibull-GA}}(x_i)} \tag{68}$$

Step 3. Selection Phase

The parent chromosome is the best solution in each iteration. Chromosomes are organized in descending order of fitness value. Only the strongest chromosomes will be sustained, while others are discarded. The selection probability for each chromosome is computed as follows:

$$p_i = \frac{\text{fit}_i}{\sum_{j=1}^{n} \text{fit}_j} \tag{69}$$

Step 4. Crossover Phase

Information from the parents is exchanged at random to produce children with distinct genetic compositions. The crossover point is randomly chosen according to the following equation:

$$x_i^{\text{new}} = rx_i + (1-r)x_j \tag{70}$$

where r is the crossover probability, and x_i and x_j represent the chromosomes.

Step 5. Mutation Phase

The chromosome information is allocated randomly within a pre-determined range during the mutation process. The mutation is expected to result in a new chromosomal breed.

$$x_i^{\text{mutated}} = x_i + \lambda(\text{rand} - 0.5) \tag{71}$$

where x_i^{mutated} is the new chromosome from the mutation stage and $\lambda \in [0, 1]$.

Step 6. Termination

The GA algorithm iterates up to 10,000 generations. If it satisfies the termination criterion, the program will stop and print the result; otherwise, it will go back to step 2 for the next iteration.

$$\text{If } i \geq 10,000 \text{ or fit} \geq \text{threshold, then stop.} \tag{72}$$

The final result of Weibull-GA is a chromosome that contains the estimated parameters of the Weibull distribution.

2.10. Goodness-of-Fit Test for Weibull Distribution

The goodness-of-fit test evaluates how well a Weibull distribution fits the empirical cumulative distribution function (ECDF) of the MIRR dataset. It aims to find the best-fitting distribution by minimizing the goodness-of-fit statistic and maximizing the p-value. In this study, the following tests are used to assess the MIRR data fit to the Weibull distribution based on various optimization methods:

$$\text{KS} = \max_i \left| F_i^{\text{Weibull}} - P_i \right| \tag{73}$$

$$\text{AIC} = 2k - 2\ln(L) \tag{74}$$

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(F_i^{\text{Weibull}} - \hat{F}_i\right)^2} \tag{75}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(F_i^{\text{Weibull}} - \hat{F}_i\right)^2}{\sum_{i=1}^{n}\left(F_i - \bar{F}\right)^2} \tag{76}$$

Coefficient of Determination (R^2): Indicates the proportion of variance explained by the Weibull model.

Algorithm 1 Parameter Estimation for Weibull Distribution

Input: Data $\{x_1, \ldots, x_n\}$; initial estimates ξ_0, β_0, η_0; temperature T_0; iterations N.
Output: Estimated parameters $\{\xi, \beta, \eta\}$.
Step 1: Initialization
Set $\mathbf{x}_0 = (\xi_0, \beta_0, \eta_0)$.
Step 2: Weibull Fitting
Fit data using: $L(\xi, \beta, \eta) = \sum_{i=1}^{n} \log \left(\frac{\beta}{\eta} \left(\frac{x_i - \eta}{\xi} \right)^{\beta - 1} e^{-\left(\frac{x_i - \eta}{\xi}\right)^{\beta}} \right)$
Step 3: Simulated Annealing (SA)
for each iteration k **do**
  Generate candidate $\mathbf{x}_k$ and compute fitness difference: $\Delta L = L(\mathbf{x}_k) - L(\mathbf{x}_0)$
  Optimization: Update candidate: $\mathbf{x}_k = \mathbf{x}_0 - \alpha \cdot \nabla L(\mathbf{x}_0)$
  if $\Delta L < 0$ or rand $< \exp\left(-\frac{\Delta L}{T}\right)$ **then**
    Accept candidate: $\mathbf{x}_{new} = \mathbf{x}_k$
  end if
  Update temperature: $T_{new} = T_0 \cdot \alpha^k$
end for
Step 4: Differential Evolution (DE)
for each candidate $\mathbf{x}_i$ **do**
  Mutation: Generate mutant vector $\mathbf{v}_i = \mathbf{x}_{r1} + F \cdot (\mathbf{x}_{r2} - \mathbf{x}_{r3})$
  Recombination: Generate trial vector

$$u_{i,j} = \begin{cases} v_{i,j} & \text{if } \text{rand}_j \leq CR \text{or } j = j_{rand} \\ x_{i,j} & \text{otherwise} \end{cases}$$

  Selection: Update $\mathbf{x}_{i+1}$:

$$\mathbf{x}_{i+1} = \begin{cases} \mathbf{u}_i & \text{if } L(\mathbf{u}_i) \geq L(\mathbf{x}_i) \\ \mathbf{x}_i & \text{otherwise} \end{cases}$$

end for
Step 5: Genetic Algorithm (GA)
for each candidate $\mathbf{x}_k$ **do**
  Compute fitness $f(\mathbf{x}_k) = -L(\mathbf{x}_k)$ and probability $P(\mathbf{x}_k) = \frac{f(\mathbf{x}_k)}{\sum_j f(\mathbf{x}_j)}$
  Selection: Select parents based on $P(\mathbf{x}_k)$
  Crossover: Generate offspring:

$$\mathbf{x}_{crossover} = \alpha \cdot \mathbf{x}_{parent1} + (1 - \alpha) \cdot \mathbf{x}_{parent2}$$

  Mutation: Apply mutation:

$$\mathbf{x}_{mut} = \mathbf{x}_{crossover} + \sigma \cdot \mathcal{N}(0, 1)$$

  Evaluate fitness and update candidate pool.
end for
Step 6: Iterative Optimization
Repeat for $N = 10,000$ iterations: Refine $\{\xi, \beta, \eta\}$.
Step 7: Statistical Evaluation
Compute evaluation metrics using Equations (75) to (78).
Output: Optimized parameters $\{\xi, \beta, \eta\}$.

3. Experimental results

In this section, we analyze the Modified Internal Rate of Return (MIRR) derived from financial reports in the Property Sector, with a specific focus on Sharia-compliant investments. The methodology and findings are outlined below.

The MIRR data, representing investment performance and share accumulation in the property sector as discussed in Section 3, was fitted to the Weibull distribution. This fitting aimed to assess investment returns and share issuance across various investment periods. To ensure accuracy, we employed multiple estimation techniques to optimize the Weibull distribution parameters, providing a comprehensive analysis of the investment landscape.

Tables 4 and 5 summarize the estimated parameters and their standard errors for each method. Figures 2 through 6 offer graphical representations of the observed and fitted Weibull distributions, providing a visual comparison of the model's fit to the MIRR data. We assessed the performance of various optimization methods, comparing their accuracy and robustness in estimating the Weibull parameters. The results highlight the superior effectiveness of metaheuristic algorithms (SA, DE, GA) compared to traditional numerical techniques (NM).

Table 4. Parameter Estimates for Weibull Distribution Models

Model	Stock size	α	λ	$\log \xi$	E(x)	$E(x)_{est}$	Var(x)	$Var(x)_{est}$
Weibull-SA	62	1.5894	2.9154	-46.3425	1.4555	1.4112	0.2986	0.2805
	124	1.4949	3.3579	-66.7069	1.3421	1.3434	0.3294	0.1698
	186	1.4719	3.3813	-99.2658	1.3220	1.3232	0.3369	0.1697
	248	1.4370	3.5462	-112.453	1.2938	1.2980	0.3446	0.1392
	310	1.4004	3.5790	-126.789	1.2615	1.2674	0.3552	0.1244
Weibull-DE	62	1.5815	2.9253	-46.1454	1.4107	1.4112	0.3096	0.2805
	124	1.4928	3.3676	-66.7105	1.3404	1.3435	0.3299	0.1698
	186	1.4747	3.3913	-99.2694	1.3247	1.3232	0.3356	0.1697
	248	1.4361	3.5564	-112.457	1.2932	1.2981	0.3446	0.1392
	310	1.4012	3.5893	-126.792	1.2625	1.2675	0.3548	0.1245
Weibull-GA	62	1.5910	2.9251	-46.1374	1.4191	1.4112	0.3059	0.2805
	124	1.4937	3.3679	-66.7092	1.3414	1.3434	0.3296	0.1698
	186	1.4810	3.3783	-99.3067	1.3301	1.3232	0.3338	0.1697
	248	1.4470	3.5532	-112.523	1.3029	1.2981	0.3411	0.1392
	310	1.4105	3.5801	-126.888	1.2706	1.2675	0.3521	0.1245
Weibull-NM	62	1.5893	2.9152	-46.1360	1.4175	1.4180	0.3069	0.2823
	124	1.4949	3.3576	-66.7060	1.3421	1.3435	0.3295	0.1698
	186	1.4720	3.3812	-99.2650	1.3220	1.3232	0.3368	0.1697
	248	1.4370	3.5463	-112.4500	1.2938	1.2981	0.3446	0.1392
	310	1.4004	3.5789	-126.7800	1.2615	1.2675	0.3552	0.1245

Table 4 provides an overview of the estimated Weibull distribution parameters over investment periods ranging from 1 to 5 years, derived through various optimization approaches. This table facilitates a comparative analysis of the parameter estimates across different numerical methods.

The Weibull distribution models (Weibull-SA, Weibull-DE, Weibull-GA, and Weibull-NM) exhibit variations in their estimated parameters concerning stock sizes (62, 124, 186, 248, 310), shape, and scale factors. For instance, within the Weibull-SA model, the shape parameter decreases from approximately 1.5894 to 1.4004 as the stock size increases from 62 to 310, while the scale parameter increases from 2.9154 to 3.5790. These trends reflect the underlying stochastic processes associated with investment behaviours. Higher shape values suggest a more rapid decline in the failure rate (i.e., risk), which may indicate a preference for riskier investments or shorter investment horizons.This is consistence with one of the basic principles of finance that stated higher risk assets tend to generate higher returns. Consequently, larger scale values, which correlate with greater projected returns or larger stock sizes, influence the dispersion and magnitude of investment returns.

An examination of the log-likelihood values at the inception and conclusion of the investment period reveals that no single model consistently outperforms the others. For example, the initial log-likelihood values for the Weibull-SA, Weibull-DE, and Weibull-GA models are -46.3425, -46.1454, and -46.1374, respectively, for a sample size of 62. At the end of the period, with a sample size of 310, the log-likelihood values are -126.789, -126.792, and -126.888, respectively. Notably, the Weibull-SA model demonstrates consistent performance across varying sample sizes. The log-likelihood values and the model fit statistics presented in Table 5 underscore the adequacy of the MIRR data in fitting Weibull distribution models. Higher log-likelihood values, such as those observed in the Weibull-SA model (ranging from -46.3425 to -126.789), suggest a better fit of the model to the data.

Table 4 also presents the true statistical characteristics of investment returns, as reflected by the precise mean and variance values. The reliability and accuracy of the models in predicting central tendency and dispersion can be assessed by comparing these true values with the estimated mean and variance values obtained from the Weibull distribution models. For instance, in the Weibull-SA model, the estimated mean values range from 1.4555 to 1.2615, and the estimated variance values range from 0.2986 to 0.3552.

Comparing the exact and estimated values across different models (e.g., Weibull-SA, Weibull-DE) and stochastic processes enables an evaluation of how various modelling techniques influence parameter estimates and model fit. Discrepancies in the predicted mean and variance values across models may highlight differences in the underlying assumptions and their impact on investment behaviour predictions.

The observed patterns in estimated mean/variance values, shape, scale, and log-likelihood across various models and stock sizes provide valuable insights into differing investment behaviours and risk tolerances. These insights are critical for risk assessment, decision-making, and predictive modelling within the field of investments.

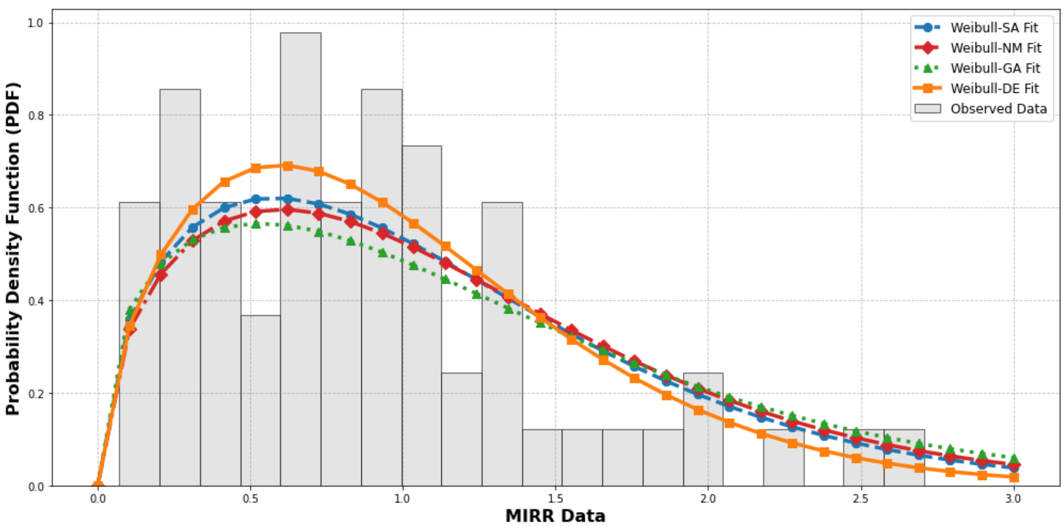

Figure 2. Comparison of Various Optimization Methods for Fitting Weibull Distributions to MIRR Data in a 1-Year Investment period

The Modified Internal Rate of Return (MIRR) data exhibits a significant degree of dispersion, necessitating the use of the Probability Density Function (PDF) for a rigorous analysis. This approach is integral for accurately estimating expected returns and assessing the risk profile associated with investment decisions. The findings of this study, as depicted in Figures 2 through 6, present histograms that are aligned with the fitted PDFs of the 2-parameter Weibull distribution. The analysis specifically focuses on Shariah-compliant enterprises over an 5-year investment horizon, from 2010 to 2015. This dataset serves as the foundation for a comprehensive evaluation of the performance of different estimation techniques within the context of financial modeling.

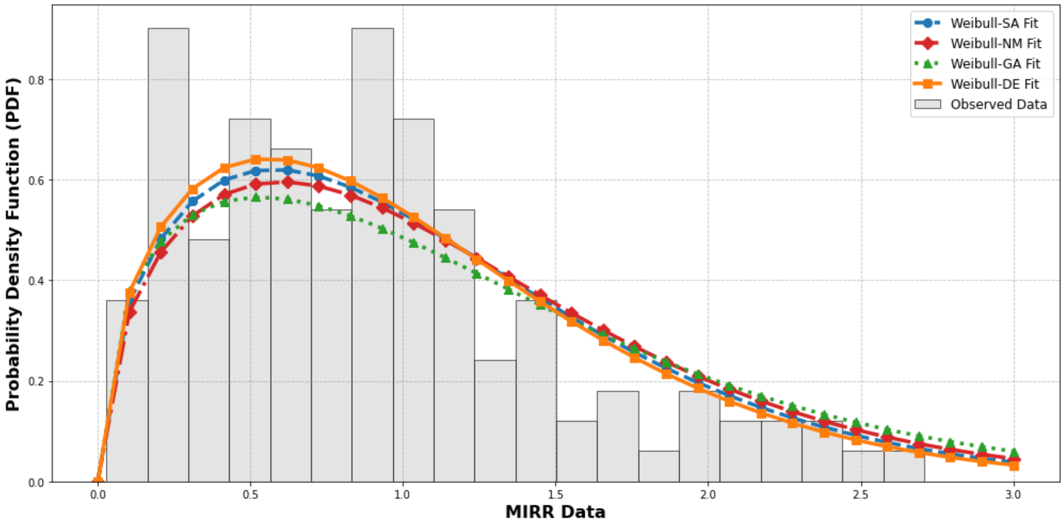

Figure 3. Comparison of Various Optimization Methods for Fitting Weibull Distributions to MIRR Data in a 2-Year Investment Period

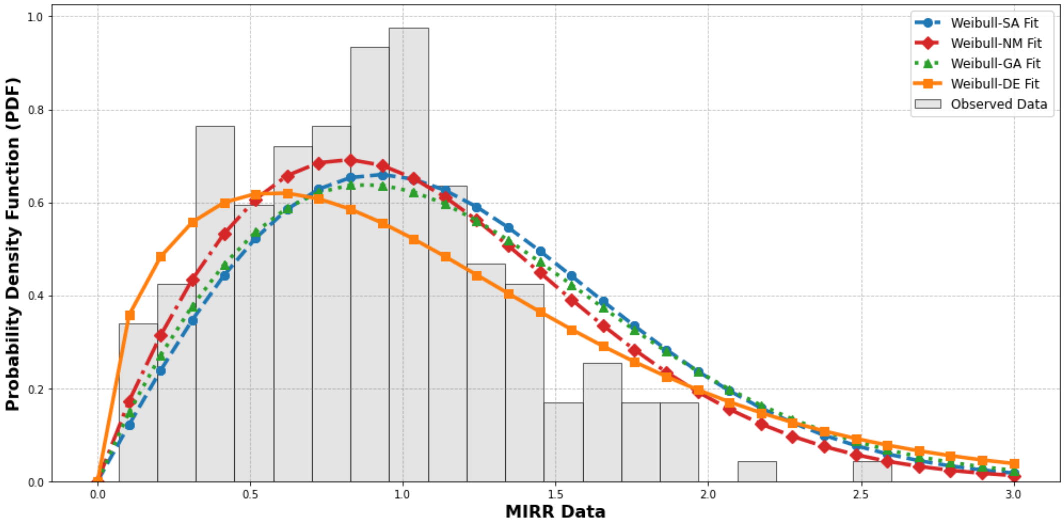

Figure 4. Comparison of Various Optimization Methods for Fitting Weibull Distributions to MIRR Data in a 3-Year Investment period.

The PDF provides a quantitative measure of the frequency with which investment returns fall within specified ranges. Typically, a normal bell curve, often plotted on a graph, is indicative of neutral market risk. However, deviations from this symmetry, as reflected in the skewness of the curve, indicate varying levels of risk-reward dynamics. Right-skewness, characterized by a leftward shift of the curve and an extended tail on the right, suggests a greater potential for upside returns. Conversely, left-skewness, marked by a leftward shift of the curve with a longer tail on the left, signals a higher likelihood of downside risk.

This study underscores the intricate nature of parameter estimation, revealing substantial differences between estimates obtained using various numerical techniques. The variations highlight the challenges and complexities inherent in financial modelling. Interestingly, the analysis of Metaheuristic Optimization Algorithms (MOAs) reveals only minimal differences in parameter estimates, suggesting a higher degree of consistency and reliability.

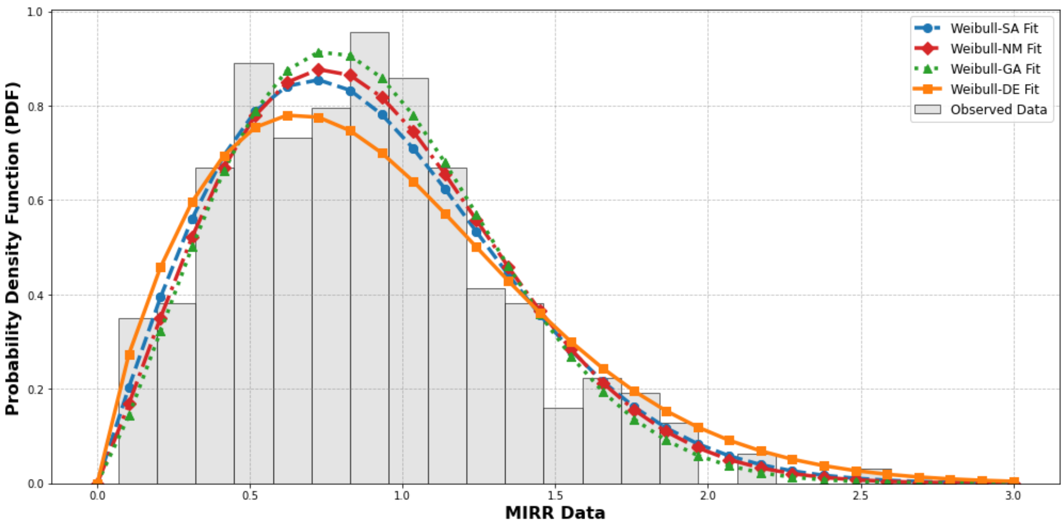

Figure 5. Comparison of Various Optimization Methods for Fitting Weibull Distributions to MIRR Data in a 4-Year Investment period

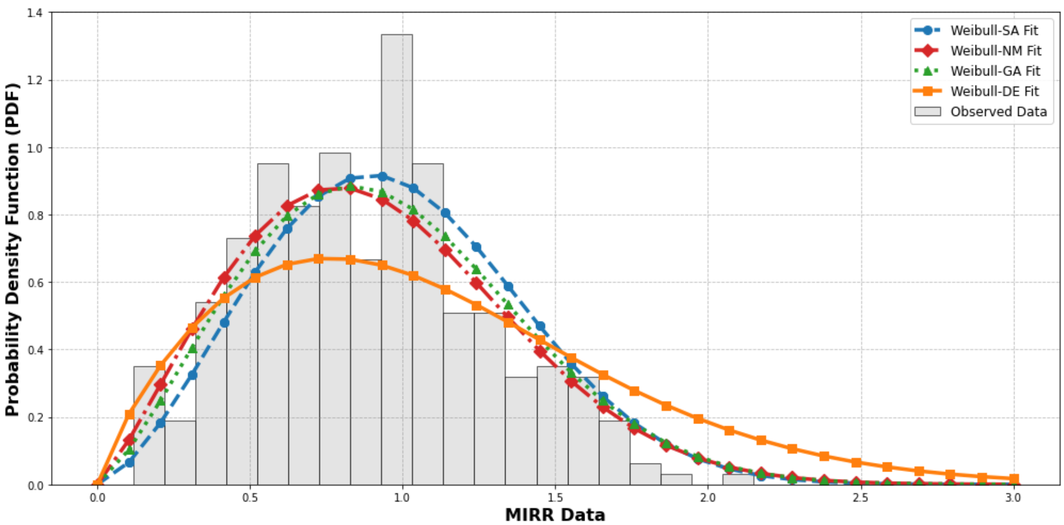

Figure 6. Comparison of Various Optimization Methods for Fitting Weibull Distributions to MIRR Data in a 5-Year Investment period

This observation emphasizes the potential efficacy of MOAs in achieving robust and dependable parameter estimates when analyzing MIRR data within financial models.

To further assess the validity and accuracy of the estimation techniques under review, Table 5 presents the results of the goodness-of-fit tests. These results provide critical insights into the effectiveness of each technique, offering a comprehensive evaluation of their performance in the context of financial modelling for the Malaysian Property Sector.

Table 5 presents the Akaike Information Criterion (AIC), R-squared (R^2), Root Mean Squared Error (RMSE), and Kolmogorov-Smirnov statistic (KS) for various estimation methods of the Weibull distribution across different parameter settings (K). The KS statistic is evaluated for $D_n > 50$ at a significance level of $\alpha = 0.05$, with a critical value of $D = 1.3581$.

Table 5. Goodness of Fit (GOT) of Weibull Distribution Based on MIRR Real Dataset

Method	K	AIC	R^2	RMSE	KS
Weibull-SA	62	-98.685	0.7609	0.1501	0.3334
	124	-137.413	0.8431	0.1192	0.2343
	186	-202.531	0.9510	0.0670	0.1367
	248	-228.906	0.9686	0.0776	0.1474
	310	-257.578	0.9639	0.0960	0.1878
Weibull-DE	62	-96.2909	0.7585	0.1575	0.7585
	124	-137.420	0.8937	0.1188	0.8437
	186	-202.539	0.9504	0.0679	0.9504
	248	-228.914	0.9388	0.0773	0.9388
	310	-257.584	0.9437	0.0963	0.9037
Weibull-GA	62	-96.2747	0.7584	0.1581	0.3350
	124	-137.418	0.8934	0.1191	0.2342
	186	-202.613	0.9493	0.0696	0.1425
	248	-229.045	0.9358	0.0809	0.1551
	310	-257.776	0.9606	0.0992	0.1958
Weibull-NM	62	-96.2734	0.7285	0.1576	0.3334
	124	-137.413	0.8431	0.1192	0.2343
	186	-202.531	0.9510	0.0670	0.1368
	248	-228.906	0.9386	0.0776	0.1474
	310	-257.578	0.9039	0.0961	0.1878

The parameters outlined in Table 5 were used to derive the goodness-of-fit (GOF) statistics detailed in Table 6. The performance of the Weibull-SA method stands out, particularly in the RMSE evaluations. For example, during the initial investment period with 62 samples, the RMSE values are 0.1501 for Weibull-SA, 0.1575 for Weibull-DE, and 0.1581 for Weibull-GA. As the sample size increases to 124, the RMSE decreases to 0.1192 for Weibull-SA, 0.1188 for Weibull-DE, and 0.1191 for Weibull-GA, indicating improved model fit.

By the time the terminal investment period is reached with 310 samples, the RMSE values further drop to 0.0960 for Weibull-SA, 0.0963 for Weibull-DE, and 0.0992 for Weibull-GA, with Weibull-SA consistently showing the lowest error margins across the board.

Weibull-SA's superior performance is further confirmed by its AIC values. During the initial investment period with 62 samples, the AIC values were -98.685 for Weibull-SA, compared to -96.2909 for Weibull-DE and -96.2747 for Weibull-GA. At the terminal investment period with 310 samples, these values are -257.578 for Weibull-SA, -257.584 for Weibull-DE, and -257.776 for Weibull-GA. These figures highlight the competitiveness and robustness of the Weibull-SA method, which also remains competitive with the Weibull-NM method across various sample sizes.

The Kolmogorov-Smirnov (KS) test results, conducted at a 5% significance level, show that the parameters derived from the NM method are particularly appropriate for short-term investment periods with smaller sample sizes, as seen with KS values like 0.3334 for 62 samples. However, Metaheuristic Optimization Algorithms (MOA) like Weibull-DE and Weibull-GA demonstrate a better fit for larger samples and longer investment durations. For example, the KS value for Weibull-SA is 0.1878 at 310 samples, compared to 0.9037 for Weibull-DE and 0.1958 for Weibull-GA.

Notably, the RMSE values for all numerical methods remain closely aligned, with Weibull-SA consistently achieving lower error rates. This positions the SA method as the most effective for predicting Weibull distribution parameters when applied to the MIRR dataset, particularly in scenarios with varying sample sizes and investment periods.

3.1. Mean and Variance Analysis of MIRR in Weibull Distribution: Estimation Techniques and Risk Insights

This study investigates the Mean and Variance of the Modified Internal Rate of Return (MIRR) within the context of the Weibull Distribution, with a focus on understanding risk and return dynamics. Figures 8 through 10 illustrate the patterns observed in both actual and estimated mean and variance values based on various optimization methods.

The analysis reveals a strong alignment between the actual and estimated parameters based on the MIRR data across different methods. However, there are noticeable discrepancies at the initial investment period, with the Weibull-SA and Weibull-NM techniques. As the sample size and investment horizon increase, the variance estimates across the optimization method tend to converge, indicating a stabilization of behaviour over time. These findings highlight the importance of using diverse optimisation techniques to predict financial data features accurately across various investment durations. Such insights are crucial for effective risk management and informed decision-making.

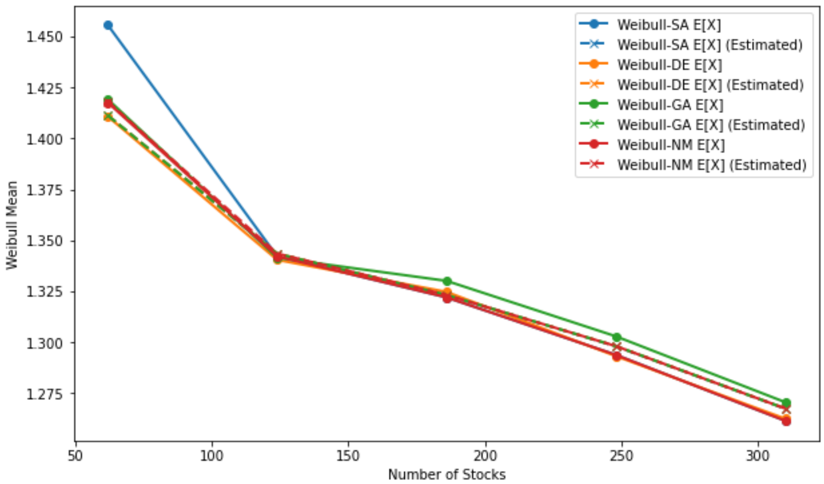

Figure 7. Comparison of Mean and Estimated Mean

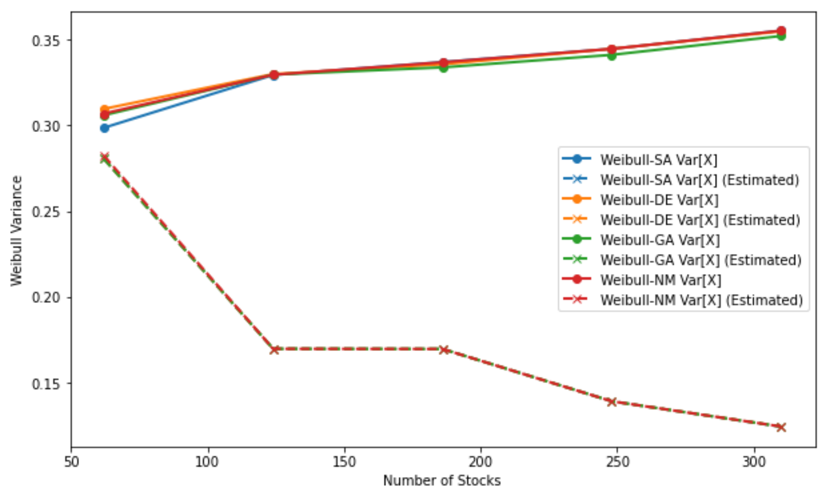

Figure 8. Comparison of Variance and Estimated variance

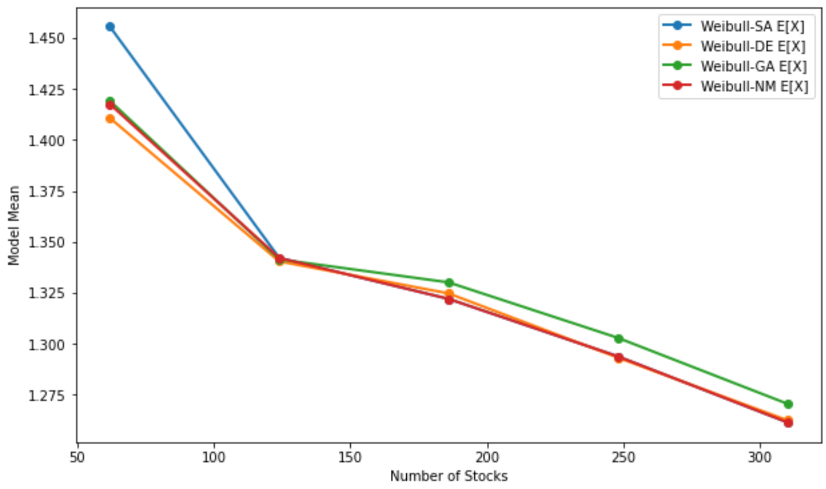

Figure 9. Mean Comparison Across Models'

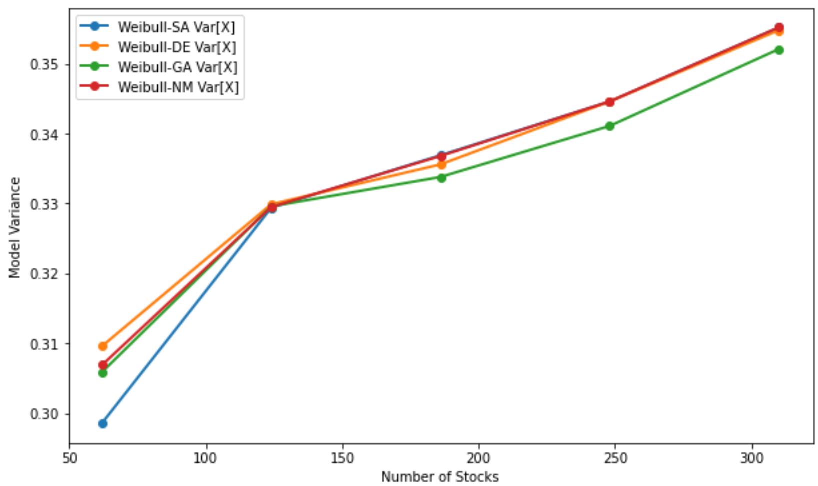

Figure 10. Variance Comparison Across Models'

3.2. Analysis and Findings

To gain a comprehensive understanding of investment return dynamics, an in-depth examination of mean-variance analysis is essential, as demonstrated in Figures 7 to 10. This method proves particularly insightful when applied to the Weibull distribution of investment return analysis data, as elaborated in Table 4. The findings presented in this study reveal a distinct declining trend in both the Mean and Variance of the investment return in a short time investment periods, contributing to the broader narrative of investment performance.

Mean-variance analysis is a crucial tool in investment decision-making, especially when risk assessment is key. It offers investors insights into balancing risk and returns, helping them make informed decisions about portfolio composition and risk management. By comparing the variance of investments with their expected returns, investors can navigate the trade-offs inherent in the investment process, aligning with Modern Portfolio Theory (MPT), which emphasizes optimizing the balance between risk and return [33], [34].

The observed decline in both mean and variance of investment returns highlights the dynamic nature of financial markets and the need for flexible investment strategies. Typically, lower variance in stocks is preferred as it indicates reduced risk. Diversification, spreading investments across a portfolio with differing variances and expected returns, mitigates risk by balancing potential losses against gains.This is also expected due to the fact that there must be a trade-off between risk and return, as such higher risk properties are expected to yield higher returns[35]. However, while investing in shares like MPS might promise long-term growth, it comes with increased risk.

The risk-return trade-off is central to investment analysis, where pursuing higher returns often involves accepting greater volatility. According to MPT [36], higher expected returns usually come with higher variance, reflecting greater uncertainty and risk. Variance, a measure of return dispersion, provides insights into the fluctuations and risk levels of a security's returns over time.

The concept of diminishing returns, discussed by Knight [37], aligns with the observed decrease in mean returns as investment size or risk level increases. For example, in the Weibull-SA model, the mean decreases from 1.4112 at a stock size of 62 to 1.2674 at a stock size of 310, indicating diminishing incremental returns.

As noted by Senchack Jr [38], [39], comparing mean values of different models at specific points, such as a stock size of 124, provides insights into their relative performance. For instance, the Weibull-NM model, with a mean of 1.3435, suggests higher returns than the Weibull-SA model at the same stock size.

Variance is equally important in risk analysis. The Weibull-NM model consistently shows higher variance than the Weibull-SA model, indicating greater volatility and risk, particularly at smaller stock sizes [40]. This variance fluctuation, which can increase with investment size as discussed by Merton [41], reflects the dynamic nature of investment risk. Investors should consider these factors carefully, conducting thorough due diligence and consulting financial experts before making investment decisions.

4. Conclusion

This study successfully developed a novel hybrid model that integrates the Modified Internal Rate of Return (MIRR) for cash flow computational analysis with the Weibull distribution for data analysis. By employing various optimization methods for parameter estimation, the study enhanced the accuracy and efficiency of financial analysis techniques. The rigorous evaluation based on goodness-of-fit criteria allowed for a comprehensive assessment of investment returns, providing valuable insights into investment performance and risk assessment.

The findings underscore the importance of mean variance analysis as a critical framework for managing risk and optimizing returns in today's volatile market environment. The study also highlights the potential of Sharia-compliant investments in MPS, offering a promising avenue for long-term profitability while adhering to ethical standards.

Looking ahead, the integration of advanced machine learning algorithms and artificial intelligence in refining parameter estimation models presents exciting opportunities for further enhancing financial analysis. Incorporating real-time market data and sentiment analysis could deepen our understanding of investment dynamics, particularly in the context of the rapidly evolving digital economy. This research lays the groundwork for future innovations in financial technology modelling, offering practical tools for investors and policymakers to navigate the complexities of modern markets.

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