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Exact travelling wave solutions of the symmetric regularized long wave (SRLW) using analytical methods

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Abstract In this article, we establish exact travelling wave solutions of the symmetric regularized long wave (SRLW) by using analytical methods. The analytical methods are: the tanh-coth method and the sech^2 method which used to construct solitary wave solutions of nonlinear evolution equations. With the help of symbolic computation, we show that aforementioned methods provide a straightforward and powerful mathematical tool for solving nonlinear partial differential equations.

Keywords Symmetric regularized long wave (SRLW); The tanh-coth method; The sech^2 method

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1. Introduction

Recently, the investigation of exact travelling wave solutions to nonlinear partial differential equations plays an important role in the study of nonlinear modelling physical phenomena. Also the study of the travelling wave solutions plays an important role in nonlinear science. Nonlinear evolution equations are widely used as models to describe complex physical phenomena and have a significant role in

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several scientific and engineering fields. A variety of powerful methods has been presented, such as Hirota's bilinear method [1], the inverse scattering transform [2], sine-cosine method [3], homotopy perturbation method [19], homotopy analysis method [5, 6], variational iteration method [7, 8, 9], the $(\frac{G'}{G})$-expansion method [10, 11], tanh-function method [12], tanhcoth method [13], Bäcklund transformation [14, 15], Exp-function method [16, 17, 18, 19, 20] and so on. In this article, we used the analytical methods to investigate the symmetric regularized long wave (SRLW) [18] in the following form

$$u_{tt} + u_{xx} + uu_{xt} + u_x u_t + u_{xxtt} = 0, \tag{1}$$

which arises in several physical applications including ion sound waves in plasma [18]. The article is organized as follows: In Sections 2 and 3 first we briefly give the steps of the methods and apply the methods to solve the nonlinear partial differential equations. In Section 4, the application of the analytical methods to the symmetric regularized long wave (SRLW) will be introduced briefly. Also a conclusion is given in Section 5. Finally some references are given at the end of this paper.

2. Basic idea of tanh–coth method

The standard tanh method is well-known analytical method which first presented by Malfliet's [21] and developed in [21, 22]. By summarizing tanh–coth method by Wazwaz [23] for a given NLPDE with independent variables $X = (x, y, z, t)$ and dependent variable u:

$$\mathcal{P}(u, u_t, u_x, u_y, u_z, u_{xx}, u_{yy}, u_{zz}, u_{xy}, u_{tt}, u_{tx}, u_{ty}, u_{tz} ...) = 0, \tag{2}$$

can be converted to on ODE

$$\mathcal{Q}(u, -cu', u', u', u', u'', ...) = 0, \tag{3}$$

which transformation $\xi = k_1 x + k_2 y - ct$ is wave variable. Also, c, k_1 and k_2 are constants to be determined later. Introducing a new independent variable

$$Y = \tanh(\mu\xi), \qquad \xi = k_1 x + k_2 y - ct, \tag{4}$$

leads to the change of derivatives

$$\frac{d}{d\xi} = \mu(1 - Y^2)\frac{d}{dY}, \tag{5}$$

$$\frac{d^2}{d\xi^2} = \mu^2(1 - Y^2)\left(-2Y\frac{d}{dY} + (1 - Y^2)\frac{d^2}{dY^2}\right), \tag{6}$$

$$\frac{d^3}{d\xi^3} = \mu^3(1-Y^2)\left((6Y^2-2Y)\frac{d}{dY} - 6Y(1-Y^2)\frac{d^2}{dY^2} + (1-Y^2)^2\frac{d^3}{dY^3}\right). \quad (7)$$

The tanh–coth method [24] admits the use of the finite expansion

$$u(\mu\xi) = S(Y) = \sum_{k=0}^{m} a_k Y^k + \sum_{k=1}^{m} b_k Y^{-k}, \quad (8)$$

where $a_k(k = 0, 2, ..., m)$, $b_k(k = 1, 2, ..., m)$ and μ are constants to be determined later, but the degree of which is generally equal to or less than $m-1$, the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq. (3). If m is not an integer, then a transformation formula should be used to overcome this difficulty. For aforementioned method, expansion (7) reduces to the standard tanh method [21] for $b_k = 0, 1 \leq k \leq m$. Substituting Eq. (7) into the ODE results is an algebraic system of equations in the powers of Y that will lead to the determination of the parameters $a_k(k = 0, 2, ..., m)$, $b_k(k = 1, 2, ..., m)$ and c. To show the efficiency of the method described in the previous part, we present some examples.

3. Basic idea of sech^2 method

We now describe the sech^2 method for the given partial differential equations. We give the detailed description of method which to use this method, we take following steps:

Step 1. For a given NLPDE with independent variables $X = (x, y, z, t)$ and dependent variable u, we consider a general form of nonlinear equation:

$$\mathcal{P}(u, u_t, u_x, u_y, u_z, u_{xx}, u_{yy}, u_{zz}, u_{xy}, u_{tt}, u_{tx}, u_{ty}, u_{tz}...) = 0, \quad (9)$$

which can be converted to on ODE

$$\mathcal{Q}(u, -cu', u', u', u', u'', ...) = 0, \quad (10)$$

which transformation $\xi = k_1x + k_2y - ct$ is wave variable. Also, c, k_1 and k_2 are constants to be determined later.

Step 2. We introduce a new independent variable as following

$$Y = \text{sech}^2(\mu\xi), \xi = k_1x + k_2y - ct, \quad (11)$$

leads to the change of derivatives in the form

$$\frac{d}{d\xi} = -2\mu Y\sqrt{1-Y}\frac{d}{dY}, \quad (12)$$

$$\frac{d^2}{d\xi^2} = -2\mu^2 Y\sqrt{1-Y}\left(\frac{3Y-2}{\sqrt{1-Y}}\frac{d}{dY} - 2Y\sqrt{1-Y}\frac{d^2}{dY^2}\right), \tag{13}$$

$$\frac{d^3}{d\xi^3} = -2\mu^3 Y\sqrt{1-Y}\left(4(1-3Y)\frac{d}{dY} + 6Y(2-3Y)\frac{d^2}{dY^2} + 4Y^2(1-Y)\frac{d^3}{dY^3}\right), \tag{14}$$

where other derivatives can be derived in a similar manner. If we use a new independent variable:

$$Y = \sec^2(\mu\xi), \xi = k_1x + k_2y - ct, \tag{15}$$

leads to the change of derivatives in the form

$$\frac{d}{d\xi} = 2\mu Y\sqrt{Y-1}\frac{d}{dY}, \tag{16}$$

$$\frac{d^2}{d\xi^2} = 2\mu^2 Y\sqrt{Y-1}\left(\frac{3Y-2}{\sqrt{Y-1}}\frac{d}{dY} - 2Y\sqrt{Y-1}\frac{d^2}{dY^2}\right), \tag{17}$$

$$\frac{d^3}{d\xi^3} = 2\mu^3 Y\sqrt{Y-1}\left(4(3Y-1)\frac{d}{dY} + 6Y(3Y-2)\frac{d^2}{dY^2} + 4Y^2(Y-1)\frac{d^3}{dY^3}\right). \tag{18}$$

The sech^2 method admits the use of a finite expansion of sech function

$$u(\mu\xi) = S(Y) = \sum_{k=0}^{m} a_k Y^k, \tag{19}$$

where a_0, $a_k(k = 1, 2, ..., m)$ and μ are constants to be determined later. but, the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq. (10).
Step 3. Substituting Eqs. (12) –(14) or Eqs. (16) –(18) into Eq. (10) with the value of m obtained in Step 2. Collecting the coefficients of Y^k $(k = 0, 1, 2, ...)$, then setting each coefficient to zero, we can get the set of over-determined nonlinear algebraic equations for $a_0, a_i(i = 1, 2, ..., n), c$ and μ with the aid of symbolic computation Maple.
Step 4. Solving the algebraic equations in Step 3, then substituting $a_0, ..., a_m, c$ in Eqs. (15) and (19).

4. Symmetric Regularized Long Wave (SRLW) Equation

4.1. Using the tanh–coth method

Considering the following Symmetric Regularized Long Wave (SRLW) equation by using the tanh–coth method, we obtain

$$u_{tt} + u_{xx} + uu_{xt} + u_xu_t + u_{xxtt} = 0, \tag{20}$$

and using the wave variable $\xi = k_1 x - ct$ reduce it to an ODE

$$c^2 u'' + {k_1}^2 u'' - ck_1 uu'' - ck_1 (u')^2 + c^2 {k_1}^2 u'''' = 0. \tag{21}$$

Integrating Eq. (21) with respect to ξ and considering the zero constants for integration, we obtain

$$({k_1}^2 + c^2)u' - ck_1 uu' + c^2 {k_1}^2 u''' = 0. \tag{22}$$

Balancing the terms that involve u''' and uu' in Eq. (22) gives

$$m + 3 = 2m + 1, \tag{23}$$

so that

$$m = 2. \tag{24}$$

The tanh–coth method allows us to use the substitution

$$u(\xi) = S(Y) = a_0 + a_1 Y + a_2 Y^2 + \frac{b_1}{Y} + \frac{b_2}{Y^2}. \tag{25}$$

Substituting Eq. (25) in to Eq. (22), with the help of Maple gives the following set of non-trivial solutions

$$a_0 = \frac{1 + c^2 - 8c^2\mu^2}{c}, \quad a_1 = 0, \quad a_2 = 12c\mu^2, \quad b_1 = 0, \quad b_2 = 0, \quad k_1 = 1, \tag{26}$$

or

$$a_0 = \frac{1 + c^2 - 8c^2\mu^2}{c}, \quad a_1 = 0, \quad a_2 = 0, \quad b_1 = 0, \quad b_2 = 12c\mu^2, \quad k_1 = 1, \tag{27}$$

or

$$a_0 = \frac{1 + c^2 - 8c^2\mu^2}{c}, \quad a_1 = 0, \quad a_2 = 12c\mu^2, \quad b_1 = 0, \quad b_2 = 12c\mu^2, k_1 = 1, \tag{28}$$

where c and μ are arbitrary constants. Substituting Eqs. (26)–(28) into expression Eq. (25), can be written as

$$u_1(x,t) = \frac{1 + c^2 - 8c^2\mu^2}{c} + 12c\mu^2 \tanh^2[\mu(x - ct)], \tag{29}$$

or

$$u_2(x,t) = \frac{1 + c^2 - 8c^2\mu^2}{c} + 12c\mu^2 \coth^2[\mu(x - ct)], \tag{30}$$

or

$$u_3(x,t) = \frac{1 + c^2 - 8c^2\mu^2}{c} + 12c\mu^2 \left(\tanh^2[\mu(x - ct)] + \coth^2[\mu(x - ct)]\right). \tag{31}$$

It is worth to point out that the following periodic solutions

$$u_4(x,t) = \frac{1+c^2+8c^2\mu^2}{c} + 12c\mu^2 \tan^2[\mu(x-ct)], \quad (32)$$

or

$$u_5(x,t) = \frac{1+c^2+8c^2\mu^2}{c} + 12c\mu^2 \cot^2[\mu(x-ct)], \quad (33)$$

or

$$u_6(x,t) = \frac{1+c^2+8c^2\mu^2}{c} + 12c\mu^2 \left(\tan^2[\mu(x-ct)] + \cot^2[\mu(x-ct)]\right). \quad (34)$$

which are the exact solutions of symmetric regularized long wave (SRLW) equation. Can be seen that the results are the same, with comparing results Darwish's and Xu's [18].

4.2. *Using the* sech^2 *method*

Considering the following Symmetric Regularized Long Wave (SRLW) equation by using the sech^2 method, and proceeding as before we obtain

$$u_{tt} + u_{xx} + uu_{xt} + u_x u_t + u_{xxtt} = 0, \quad (35)$$

and using the wave variable $\xi = k_1 x - ct$ reduce it to an ODE

$$c^2 u'' + {k_1}^2 u'' - ck_1 uu'' - ck_1 (u')^2 + c^2 {k_1}^2 u'''' = 0. \quad (36)$$

Integrating Eq. (36) with respect to ξ and considering the zero constants for integration, we obtain

$$({k_1}^2 + c^2)u' - ck_1 uu' + c^2 {k_1}^2 u''' = 0, \quad (37)$$

for simplicity suppose $k_1 = 1$. By a similar derivation as illustrated in the previous section, we obtain

$$m = 2. \quad (38)$$

Therefore by use of the sech^2 method, we may choose a solution in the form

$$u(\xi) = S(Y) = a_0 + a_1 Y + a_2 Y^2, \; a_2 \neq 0. \quad (39)$$

Substituting Eq. (39) in to Eq. (37), and by using the well-known software Maple, and equating the coefficients of the powers Y, we then get the following algebraic relations:

$$-2c^2 a_2 - 2a_2 + ca_1^2 + 12c^2\mu^2 a_1 + 2ca_0 a_2 - 32c^2\mu^2 a_2 = 0, \quad (40)$$

$$60c^2\mu^2 a_2 + 3ca_1 a_2 = 0,$$

$-a_1 - 4c^2\mu^2 a_1 - c^2 a_1 + ca_0 a_1 = 0.$

By using explicitly Eqs. (39) and (40) we solve this system with the aid of the Maple Package, we obtain the system of following results:

$$a_0 = \frac{1}{60}\frac{(9+36\mu^2)a_2^2 + 400\mu^4}{a_2\mu^2}, \quad a_1 = -3a_2, \quad a_2 = a_2, \quad c = \frac{3}{20}\frac{a_2}{\mu^2}, \quad \mu = \mu. \tag{41}$$

Substituting Eq. (41) in Eq. (39) along with (11), we obtain exact travelling wave solution for Eq. (35) of the form:

$$u(x,t) = \frac{1}{60}\frac{(9+36\mu^2)a_2^2 + 400\mu^4}{a_2\mu^2} - 3a_2\mathrm{sech}^2\mu\left(\mathrm{x} - \frac{3}{20}\frac{\mathrm{a}_2}{\mu^2}\mathrm{t}\right) + a_2\mathrm{sech}^4\mu\left(\mathrm{x} - \frac{3}{20}\frac{\mathrm{a}_2}{\mu^2}\mathrm{t}\right). \tag{42}$$

If we choose the following solution forms of Eqs. (15)–(18) and insert them into Eq. (37), equating the coefficients of the powers Y, then we get the following algebraic relations:

$$2a_2 - 2ca_0a_2 - ca_1^2 + 2c^2a_2 - 32c^2\mu^2a_2 + 12c^2\mu^2a_1 = 0, \tag{43}$$

$$60c^2\mu^2a_2 - 3ca_1a_2 = 0,$$

$$a_1 + c^2a_1 - 4c^2\mu^2a_1 - ca_0a_1 = 0.$$

By the same manipulation as illustrated above, we obtain

$$a_0 = -\frac{1}{60}\frac{(9-36\mu^2)a_2^2 + 400\mu^4}{a_2\mu^2}, \quad a_1 = -3a_2, \quad a_2 = a_2, \quad c = -\frac{3}{20}\frac{a_2}{\mu^2}, \quad \mu = \mu. \tag{44}$$

where c and μ are arbitrary constants. Substituting Eq. (44) in Eq. (39) along with (15), we obtain exact travelling wave solution for Eq. (35) of the form:

$$u(x,t) = \frac{1}{60}\frac{(9-36\mu^2)a_2^2 + 400\mu^4}{a_2\mu^2} - 3a_2\sec^2\mu\left(\mathrm{x} + \frac{3}{20}\frac{\mathrm{a}_2}{\mu^2}\mathrm{t}\right) + a_2\sec^4\mu\left(\mathrm{x} + \frac{3}{20}\frac{\mathrm{a}_2}{\mu^2}\mathrm{t}\right). \tag{45}$$

which is the exact solution of symmetric regularized long wave (SRLW) equation. Can be seen that the results are the same, with comparing results Xu's [18].

5. Conclusion

In this article we investigated the symmetric regularized long wave (SRLW) equation by the analytical methods. Obtained the solitary wave and periodic wave solutions by the tanh-coth method and the sech^2 method. These methods have been successfully applied to obtain some new generalized solitonary solutions to the symmetric regularized long wave (SRLW) equation. The tanh-coth method and the sech^2 method are more powerful in searching for exact solutions of NLPDEs.

Some of these results are in agreement with the results reported specially by Xu's [18]. Also, new results are formally developed in this article. It can be concluded that the these methods are a very powerful and efficient technique in finding exact solutions for wide classes of problems.

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