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Some Estimation Approaches of Intensities for a Two Stage Open Queueing Network

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Abstract In this paper we propose a consistent and asymptotically normal estimator (CAN) for intensity parameters ρ_1 & ρ_2 for a queueing network with distribution-free inter-arrival and service times. Using this estimator and its estimated variance, a $100(1-\alpha)\%$ asymptotic confidence interval for intensities is constructed. Variance-stabilized bootstrap-t, Bayesian bootstrap, Percentile bootstrap are also applied to develop the confidence intervals for intensities. A comparative analysis is conducted to demonstrate performances of the confidence intervals of intensities for a queueing network with short run.

Keywords Coverage percentage; Relative coverage; Variance-stabilized bootstrap-t; Bayesian bootstrap; Percentile bootstrap; Slutsky's theorem

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1. Introduction

Queueing network is characterized by one or more sources of job arrivals and corresponding one or more sinks that absorb jobs departing from the network. Consider the two-stage open queueing network shown in Figure-1.
The system consists of two nodes with respective service rates μ_1 and μ_2. The external arrival rate is λ. Burke (1956) has shown that the output of an M/M/1

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queue is also Poisson with rate λ. Traffic intensities are defined as the ratios

$$\rho_1 = \frac{\lambda}{\mu_1}, \quad \rho_2 = \frac{\lambda}{\mu_2} \tag{1}$$

where $1/\lambda$ represent mean inter-arrival time and $1/\mu_1, 1/\mu_2$ denotes mean service times at node-1 and node-2 respectively. Intensity parameters ρ_1 and ρ_2 can be interpreted as expected number of arrivals per mean service time in the limit, an important parameters called utilization factors that measures the average use of the service facility (Gross and Harris (1998)). The condition for stability of the system is that ρ_1 and ρ_2 must be less than unity.

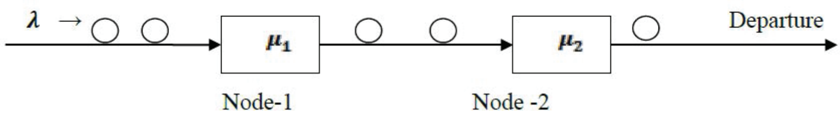

Figure-1: Two-stage open queueing network.

Jackson (1957) showed that the product form solution also applies to open network of Markovian queues with feedback, also Jacksons theorem states that each node behaves like an independent queue. Disney (1975) introduces basic properties of queueing networks. Thiruvaiyaru, Basawa and Bhat (1991) established maximum likelihood estimators of the parameters of an open Jackson network, and their joint asymptotic normality. Thiruvaiyaru and Basawa (1996) considered the problem of estimation for the parameters in a Jacksons type queueing network with the arrival at each node following renewal process and service time distribution being arbitrary. Open queueing networks are useful in studying the behavior of computer communication networks (Kleinrock, 1976).

Efron (1979, 1982, and 1987) originally developed and proposed the bootstrap to estimate the sampling distribution of any statistic. Today the bootstrap becomes the most powerful nonparametric estimation procedure. Based upon the bootstrap resampling technique, most statisticians utilize the standard bootstrap (SB), percentile bootstrap (PB), and bias-corrected and accelerated bootstrap (BCaB) approaches to produce confidence intervals for practical problems. Rubin (1981) presented the Bayesian bootstrap (BB) technique of resampling. Ke and Chu (2006) proposed a consistent and asymptotically normal estimator of intensity for a queueing system with distribution-free inter-arrival and service times. Ke and Chu (2009) constructed confidence intervals of intensity for a queueing system, which are based on different bootstrap methods.

In this paper we propose different types of interval estimations for intensity parameters ρ_1, ρ_2 for a two- stage open queueing network with distribution-free interval and service times. Also, numerical simulation study is conducted to demonstrate performances of the interval estimation approaches for a two-stage open queueing network with short run. All simulation results are shown by appropriate tables for illustrating performances of all estimation approaches. Finally some conclusions are made.

2. Nonparametric Statistical Inference for Estimating Intensity Parameters

Let $(X_i, Y_i, i = 1, 2)$ be nonnegative random variables representing respectively inter-arrival times and service times at node-1 and node-2 of a queueing network.The random variables $(X_i, Y_i, i = 1, 2)$ of node-1 and node-2 are independent. Consider $(X_{ij}, Y_{ij}, i = 1, 2, j = 1, 2, \cdots, n)$ is a random sample drawn from $(X_i, Y_i, i = 1, 2)$ for j^{th} customer at i^{th} node. The intensities are defined as follows:

$$\rho_1 = \frac{\mu_{Y_1}}{\mu_{X_1}} \quad \text{and} \quad \rho_2 = \frac{\mu_{Y_2}}{\mu_{X_2}}, \tag{2}$$

where μ_{X_1}, μ_{X_2} denote the mean inter-arrival times at node-1 and node-2 respectively. Also μ_{Y_1}, μ_{Y_2} denote the mean service times at node-1 and node-2 respectively. Let $(\overline{X}_i, \overline{Y}_i, i = 1, 2)$ be the sample means of $(X_{ij}, Y_{ij}, i = 1, 2, j = 1, 2, \cdots, n)$ respectively. According to the Strong Law of Large Numbers (see Rousses, 1997, p.196), we know that $(\overline{X}_i, \overline{Y}_i, i = 1, 2)$ are strongly consistent estimators of $(\mu_{X_i}, \mu_{Y_i}, i = 1, 2)$ respectively. Thus strongly consistent estimators of intensities are given by

$$\hat{\rho}_i = \frac{\overline{Y}_i}{\overline{X}_i}, \quad i = 1, 2. \tag{3}$$

The true distributions of $(X_i, Y_i, i = 1, 2)$ are not often known in practice so the exact distributions of $\hat{\rho}_i, i = 1, 2$ cannot be derived. But under the assumption that X_i and Y_i being independent, the asymptotic distributions of $\hat{\rho}_i, i = 1, 2$ can be developed by the following procedure. By Slutsky's theorem (Hogg & Craig, 1995), we have

$$\sqrt{n} \begin{pmatrix} \hat{\rho}_1 - \rho_1 \\ \hat{\rho}_2 - \rho_2 \end{pmatrix} \xrightarrow{D} N_2 \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \right)$$

where $\sigma_i^2 = (\mu_{X_i}^2 \sigma_{Y_i}^2 + \mu_{Y_i}^2 \sigma_{X_i}^2)/\mu_{X_i}^4, i = 1, 2$ and $\xrightarrow{D}$ denotes convergence in distribution. Now we can estimate σ_i^2 as

$$\hat{\sigma}_i^2 = (\overline{X}_i^2 S_{Y_i}^2 + \overline{Y}_i^2 S_{X_i}^2)/\overline{X}_i^4, \quad i = 1, 2$$

where

$$S_{x_i}^2 = \frac{1}{n}\sum_{j=1}^{n}(X_{ij} - \overline{X}_i)^2, \;\; S_{Y_i}^2 = \frac{1}{n}\sum_{j=1}^{n}(Y_{ij} - Y_i)^2, \;\; i = 1, 2$$

Then $\hat{\sigma}_i^2, i = 1, 2$ is a strongly consistent estimator of $\sigma_i^2, i = 1, 2$. Again applying the Slutsky's theorem we have,

$$\sqrt{n}\begin{pmatrix}(\hat{\rho}_1 - \rho_1)/\hat{\sigma}_1 \\ (\hat{\rho}_2 - \rho_2)/\hat{\sigma}_2\end{pmatrix} \xrightarrow{D} N_2\left(\begin{pmatrix}0 \\ 0\end{pmatrix}, \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix}\right)$$

Thus $\hat{\rho}_i, i = 1, 2$ is a strongly consistent and asymptotically normal (CAN) estimator with approximate variances $\hat{\sigma}_i^2/n, i = 1, 2$.

3. Consistent and Asymptotically Normal (CAN) Confidence Intervals

Using CAN estimators $\hat{\rho}_i, i = 1, 2$ and its associated approximate variances $\hat{\sigma}_i^2/n, i = 1, 2$, we construct confidence intervals for intensities $\rho_i, i = 1, 2$ for a distribution-free two-stage open queueing network. Let z_α be the upper α^{th} quantile of the standard normal distribution. Thus we have $100(1 - \alpha)\%$ confidence intervals for $\rho_i, i = 1, 2$

$$\left(\hat{\rho}_i \pm z_{a/2}\hat{\sigma}_i/\sqrt{n}\right), \;\; i = 1, 2. \tag{4}$$

4. Variance-stabilized Bootstrap-*t* (VST) Confidence Intervals

According to the bootstrap procedure, a simple random sample $(X_{ij}^*, Y_{ij}^*, i = 1, 2, j = 1, 2, \cdots, n)$ called a bootstrap sample is taken from the empirical distribution function of $(X_{ij}, Y_{ij}, i = 1, 2; j = 1, 2, \cdots, n)$. Thus bootstrap estimate for intensity $\rho_i, i = 1, 2$ can be calculated as

$$\hat{\rho}_i^* = \frac{\overline{y}_i^*}{\overline{x}_i^*}, i = 1, 2. \tag{5}$$

. The above resampling process is repeated N times. The N bootstrap estimates $\hat{\rho}_{i1}^*, \hat{\rho}_{i2}^*, \cdots, \hat{\rho}_{iN}^*, i = 1, 2$ are computed from the bootstrap resample.Let $\hat{\rho}_i, i = 1, 2$ be strongly consistent and asymptotically normal estimator with approximate variances $\hat{\sigma}_i^2/n, i = 1, 2$ and consider $\hat{\sigma}_i = \phi(\hat{\rho}_i)$. To find a transformation $f(\hat{\rho}_i)$ such that $Var(f(\hat{\rho}_i)) \approx$ constant,by the first order Taylor series expansion:

$$f(\hat{\rho}_i) \approx f(\rho_i) + (\hat{\rho}_i - \rho_i)f'(\rho_i) \Rightarrow [f(\hat{\rho}_i) - f(\rho_i)]^2 \approx (\hat{\rho}_i - \rho_i)^2(f'(\rho_i))^2, \;\; i = 1, 2.$$

Taking expectations on both sides, we get:

$$Var[f(\hat{\rho}_i)] \approx Var(\hat{\rho}_i)(f'(\rho_i))^2 = (\phi(\rho_i))^2(f'(\rho_i))^2, \;\; i = 1, 2.$$

Now consider $f(\hat{\rho}_i) = \sqrt{n}\log(\phi(\hat{\rho}_i)), i = 1,2$ is the variance-stabilizing transformation. Then we have,

$$V[f(\hat{\rho}_i)] \approx \left(\frac{\sqrt{n}}{\phi(\hat{\rho}_i)}\right)^2 \; Var[\hat{\rho}_i] = \left(\frac{\sqrt{n}}{\hat{\sigma}_i}\right)^2 \; Var[\hat{\rho}_i] = \frac{n}{\hat{\sigma}_i^2}\frac{\hat{\sigma}_i^2}{n} = 1, i = 1,2.$$

Here we consider N bootstrap estimates $\hat{\rho}^*_{i1}, \hat{\rho}^*_{i2}, \cdots, \hat{\rho}^*_{iN}, i = 1,2$ computed from the bootstrap resample. We obtain

$$\theta^*_{ij} = (\sqrt{n}\log(\hat{\rho}^*_{ij}) - \sqrt{n}\log(\hat{\rho}_i)), i = 1,2, j = 1,2,\cdots, N.$$

Thus we have $100(1-\alpha)\%$ Variance-stabilized Bootstrap-t(VST) confidence interval for $\rho_i, i = 1,2$ are

$$\left(e^{\log(\hat{\rho}_i) - \frac{1}{\sqrt{n}}\hat{v}_i t_{1-\alpha/2}}, \; e^{\log(\hat{\rho}_i) - \frac{1}{\sqrt{n}}\hat{v}_i t_{\alpha/2}}\right) \tag{6}$$

where $\hat{v}_i t_{\alpha/2}$ and $\hat{v}_i t_{1-\alpha/2}$ are $(\alpha/2)^{th}$ and $(1-\alpha/2)^{th}$ percentile of the random sample $\theta^*_{i1}, \theta^*_{i2}, \cdots, \theta^*_{iN}, i = 1,2$.

5. Bayesian Bootstrap (BB) Confidence Intervals

Each Bayesian Bootstrap (BB) replication generates a posterior probability for each $X_{ij}, i = 1,2, j = 1,2,\cdots,n$. One BB replication is generated by drawing $n-1$ uniform $(0,1)$ random numbers $r_1, r_2, \cdots, r_{n-1}$ ordering them, and calculating the gaps $w_j = r_{(j)} - r_{(j-1)}, j = 1,2,\cdots,n$, where $r_{(0)} = 0$ and $r_{(n)} = 1$. Then $w_i = (w_{i1}, w_{i2}, \cdots, w_{in}), i = 1,2$ is the vector of probabilities attached to the inter-arrival data $X_{ij}, i = 1,2, j = 1,2,\cdots,n$. Considering all BB replications gives the BB distribution of the distribution of X_i and thus of any parameter of this distribution. Hence for $\mu_{x_i}, i = 1,2$ (the mean of X_i) in each BB replication we calculate $\mu_{x_i}, i = 1,2$ as if w_{ij} were the probability that $X_i = x_{ij}$ that is, we calculate $\overline{X}_i^{**} = \sum_{j=1}^{n} w_{ij}x_{ij}, i = 1,2$. The distribution of the values of $\overline{X}_i^{**}$ over all BB replications is the BB distribution of μ_{X_i}. Also, generating a vector of probabilities $v_i = (v_{i1}, v_{i2}, \cdots, v_{in}), i = 1,2$ attached to the service time data values $Y_{ij}, i = 1,2, j = 1,2,\cdots,n$ in a BB replication, and we calculate $\overline{Y}_i^{**} = \sum_{j=1}^{n} v_{ij}y_{ij}$ for μ_{Y_j} (the mean of Y_i). An estimate of intensity ρ_i can be calculated from BB replications as $\hat{\rho}_i^{**} = \frac{\overline{Y}_i^{**}}{\overline{X}_i^{**}}, i = 1,2$, where $\hat{\rho}_i^{**}, i = 1,2$ is called a Bayesian bootstrap estimate of $\rho_i, i = 1,2$. The above BB process can be repeated N times. The NBB estimates $\hat{\rho}^*_{i1}, \hat{\rho}^*_{i2}, \cdots, \hat{\rho}^*_{iN}, i = 1,2$ can be computed from the BB replications. Averaging the NBB estimates, we obtain that $\hat{\rho}'_{BB}(i) = \frac{1}{N} \sum_{j=1}^{N} \hat{\rho}^{**}_{ij}, i = 1, 2$ is the BB estimate of $\rho_i, i = 1, 2$. Also the standard deviation of $\hat{\rho}_i$ can be estimated by

$$sd(\hat{\rho}'_{BB}(i)) = \left\{ \frac{1}{N-1} \sum_{j=1}^{N} (\hat{\rho}^{**}_{ij} - \hat{\rho}'_{BB}(i))^2 \right\}^{1/2}, \quad i = 1, 2.$$

Applying the asymptotic normality of $\hat{\rho}_i, i = 1, 2$, we have $100(1-\alpha)\% BB$ confidence interval for $\rho_i, i = 1, 2$ are

$$\left(\hat{\rho}_i \pm z_{\alpha/2} sd(\hat{\rho}'_{BB}(i))\right), \quad i = 1, 2. \tag{7}$$

6. Percentile Bootstrap (PB) Confidence Intervals

Now call $\hat{\rho}^*_{i1}, \hat{\rho}^*_{i2}, \cdots, \hat{\rho}^*_{iN}, i = 1, 2$ the bootstrap distribution of $\hat{\rho}_i, i = 1, 2$. Let $\hat{\rho}^*_i(1), \hat{\rho}^*_i(2), \cdots, \hat{\rho}^*_i(N), i = 1, 2$ be the order statistics of $\hat{\rho}^*_{i1}, \hat{\rho}^*_{i2}, \cdots, \hat{\rho}^*_{iN}, i = 1, 2$. Then utilizing the $100(\alpha/2)^{th}$ and $100(1-\alpha/2)^{th}$ percentage points of the bootstrap distribution, $100(1-\alpha)\%$ PB confidence interval for $\rho_i, i = 1, 2$ are obtained as

$$\left(\hat{\rho}^*_i\left(\left[N\left(\frac{\alpha}{2}\right)\right]\right), \; \hat{\rho}^*_i\left(\left[N\left(1-\frac{\alpha}{2}\right)\right]\right)\right), \quad i = 1, 2 \tag{8}$$

where $[x]$ denotes the greatest integer less than or equal to x.

7. Simulation Study

A numerical simulation study was undertaken to evaluate performance of the various interval estimation approaches mentioned above for a two-stage open queueing network with short run. It is observed that most statisticians assess performances of interval estimations in terms of coverage percentages or average lengths of confidence intervals. However, through simulation study in the research work, we find that larger coverage percentages of confidence interval may often be due to wider standard deviation of interval estimation methods. Moreover, narrower confidence intervals may often lead to smaller coverage percentages. Hence, both coverage percentage and average length are not efficient for appraising interval estimation methods. In order to overcome above two shortcomings, we consider a measure, named relative coverage, to evaluate performances of interval estimation methods.

Table-1 : Different levels of intensity parameters considered in the simulation study

$\rho_1 < \rho_2$	$\rho_1 > \rho_2$
(1) Low=0.1 and Moderate=0.5	(1) Moderate=0.5 and Low=0.1
(2) Low=0.1 and High=0.9	(2) High=0.9 and Low=0.1
(3) Moderate=0.5 and High=0.9	(3) High=0.9 and Moderate=0.5

Relative coverage is defined as the ratio of coverage percentage to average length of confidence interval. Larger relative coverage implies the better performances of the corresponding confidence intervals. In order to reach this goal, we not only set a continuous distribution with mean $1/\lambda$ on inter-arrival time X_1 and X_2 but also assume a continuous distribution with mean $1/\mu_1$ on the service time Y_1 at node-1 and that of $1/\mu_2$ on Y_2 at node-2.

Table-2 : Different queueing network models simulated for study

Queueing Networks type	Models simulated	C.V. of inter-arrival time for node-1	C.V. of inter- arrival time for node-2	C.V. of service time for node-1	C.V. of service time for node-2
M/G/1 to	$M/E_4/1$ to $E_4/M/1$	1	1/2	1/2	1
G/M/1	$M/H_4^{Pe}/1$ to $H_4^{Pe}/M/1$	1	> 1	> 1	1
G/G/1 to	$E_4/H_4^{Pe}/1$ to $H_4^{Pe}/E_4/1$	1/2	> 1	> 1	1/2
G/G/1	$E_4/H_4^{Po}/1$ to $H_4^{Po}/E_4/1$	1/2	< 1	< 1	1/2

For each level of $\rho_i, i = 1, 2$ random samples of inter-arrival times and service times $(X_{ij}, Y_{ij}, i = 1, 2, j = 1, 2, \cdots, n$ are drawn from $(X_i, Y_i, i = 1, 2)$. Next $N = 1000$ bootstrap resamples each of size $n = 10, 20\&25$ are drawn from the original samples, as well as N=1000 BB replications are simulated for the original samples. According to equations (4) to (8) we obtain CAN, VST, BB and PB confidence intervals of intensities ρ_1 and ρ_2 with confidence level 90%. The above simulation process is replicated $N = 1000$ times and we compute coverage percentages, average lengths and relative coverage of the above mentioned confidence intervals. We utilize a PC Dual Core and apply Matlab 7.0.1 to accomplish all simulations. Here C.V. represents coefficient of variation corresponding to the inter-arrival/service time distribution, M represents an exponential distribution, E_4 a 4-stage Erlang distribution, H_4^{Pe} a 4-stage hyper-exponential distribution and H_4^{Po} a 4-stage hypo-exponential distribution.

Table-7 : Performances of the estimation approaches of intensities ρ_1 and ρ_2 under various Queueing Networks

Queueing Network Type	Queuing Network simulated	Queueing Network with greater relative coverage	Intensity Parameters	Estimation approach with greatest relative coverage $n = 10$	$n = 20$	$n = 25$
$M/G/1$ to $G/M/1$	$M/E_4/1$to $E_4/M/1$ and $M/E_4^{Pe}/1$ to $H_4^{Pe}/M/1$	$M/E_4/1$ to $E_4/M/1$	$\rho_1 = 0.1$ $\&\rho_2 = 0.5$	VST BB	CAN BB	CAN BB
			$\rho_1 = 0.1$ $\&\ \rho_2 = 0.9$	CAN BB	CAN BB	VST BB
			$\rho_1 = 0.5$ $\&\rho_2 = 0.1$	VST BB	CAN BB	VST BB
			$\rho_1 = 0.5$ $\&\rho_2 = 0.9$	VST BB	CAN BB	VST PB
			$\rho_1 = 0.9$ $\&\rho_2 = 0.1$	CAN BB	VST PB	VST BB
			$\rho_1 = 0.9$ $\&\rho_2 = 0.5$	VST BB	VST BB	BB PB
$G/G/1$ to $G/G/1$	$E_4/H_4^{pe}/1$ to $H_4^{Pe}/E_4/1$ and $E_4/H_4^{Po}/1$ to $H_4^{Po}/E_4/1$	$E_4/H_4^{Pe}/1$ to $H_4^{Pe}/E_4/1$	$\rho_1 = 0.1$ $\&\rho_2 = 0.5$	BB BB	BB BB	BB BB
			$\rho_1 = 0.1$ $\&\ \rho_2 = 0.9$	BB BB	BB BB	BB BB
			$\rho_1 = 0.5$ $\&\rho_2 = 0.1$	BB BB	BB BB	BB BB
			$\rho_1 = 0.5$ $\&\rho_2 = 0.9$	BB BB	BB BB	BB BB
			$\rho_1 = 0.9$ $\&\rho_2 = 0.1$	BB BB	BB BB	BB BB
			$\rho_1 = 0.9$ $\&\rho_2 = 0.5$	BB BB	BB BB	BB BB

Based on the above mentioned interval estimation approaches the coverage percentage, average lengths and relative coverage of intensities ρ_1 and ρ_2 are shown in Tables 3 to 6 (see Appendix) for queuing network models (presented in Table 2) with short run. According to the simulation results in Tables 3 to 6, we find that average lengths are decreases but both coverage percentages and relative coverage are increases with sample size n. Also we observe that the coverage percentage can approaches to 90 % when n increases to 25.

Based on Table 7, we note that:

(1) Under $M/G/1$ to $G/M/1$ model the confidence intervals corresponding to queueing network with inter-arrival distribution and service time distribution of small CV (< 1) have greater relative coverage than those of large CV (> 1) for intensities ρ_1 and ρ_2. The estimation approaches Consistent and Asymptotically Normal estimator, Variance-stabilized Bootstrap-t and Bayesian bootstrap has the greatest relative

coverage. Also the confidence intervals of $M/E_4/1$ to $E_4/M/1$ shows the greatest relative coverage for ρ_1 and ρ_2.

(2) Under $G/G/1$ to $G/G/1$ models the confidence interval corresponding to queueing network models with inter-arrival distribution and service time distribution of large CV(>1) have greatest relative coverage than those of small CV(<1) for intensities ρ_1 and ρ_2. The estimation approach Bayesian bootstrap has the greatest relative coverage. Also the confidence intervals of $E_4/H_4^{Pe}/1$ to $H_4^{Pe}/E_4/1$ shows the greatest relative coverage for ρ_1 and ρ_2.

(3) Average lengths are decreases and relative coverage increases with n increases for ρ_1 and ρ_2.

Based upon our additional simulation study (not be presented), all the above mentioned approaches perform almost equally well on the interval estimation for intensities ρ_1 and ρ_2 when the sample size n is sufficiently large.

8. Conclusions

This paper provides the interval estimations of intensities ρ_1 and ρ_2 for two-stage open queueing network with short run data. Different estimation approaches CAN, VST, BB and PB are applied to produce confidence intervals for intensities ρ_1 and ρ_2. The relative coverage is adopted to understand, compare and assess performance of the resulted confidence intervals. The simulation results imply that the CAN, VST and BB method has the best performance for $M/G/1$ to $G/M/1$ queueing network and under $G/G/1$ to $G/G/1$ queueing networks, the estimation approach BB out performs. The above mentioned approaches are easily applied to practical queueing network such as all types of open, closed, mixed queueing networks as well as cyclic, retrial queueing models. Further research may consider investigations of other characteristics of a queueing network with small sample data by using the different estimation approaches.

Appendix:

Table-3 : Simulation results of coverage percentage, average lengths and relative coverage for 90% confidence intervals of queueing Network $M/E_4/1$ to $E_4/M/1$

Intensity Parameters	Estimation Approches	Coverage Percentages			Average Lengths			Relative Coverage		
		$n=10$	$n=20$	$n=25$	$n=10$	$n=20$	$n=25$	$n=10$	$n=20$	$n=25$
	CAN1	0.857	0.873	0.900	0.121	0.084	0.075	7.095	**10.346**	**12.057**
	CAN2	0.859	0.879	0.889	0.569	0.402	0.368	1.509	2.186	2.415
	VST1	0.838	0.856	0.886	0.118	0.083	0.074	**7.125**	10.297	12.009
$\rho_1 = 0.1$	VST2	0.879	0.878	0.879	0.649	0.425	0.383	1.354	2.065	2.295
&	BB1	0.859	0.873	0.897	0.125	0.085	0.075	6.893	10.286	12.008
$\rho_2 = 0.5$	BB2	0.837	0.867	0.881	0.532	0.389	0.358	**1.574**	**2.231**	**2.463**
	PB1	0.828	0.855	0.872	0.136	0.089	0.078	6.074	9.606	11.247
	PB2	0.860	0.880	0.886	0.552	0.395	0.362	1.559	2.226	2.447
	CAN1	0.861	0.898	0.880	0.121	0.084	0.074	**7.131**	**10.749**	11.911
	CAN2	0.835	0.853	0.876	1.008	0.714	0.652	0.829	1.195	1.343
	VST1	0.831	0.882	0.877	0.117	0.082	0.073	7.087	10.714	**11.999**
$\rho_1 = 0.1$	VST2	0.861	0.862	0.868	1.154	0.754	0.681	0.746	1.143	1.274
&	BB1	0.867	0.898	0.874	0.125	0.084	0.074	6.950	10.688	11.827
$\rho_2 = 0.9$	BB2	0.817	0.848	0.869	0.939	0.690	0.634	**0.870**	**1.229**	**1.371**
	PB1	0.830	0.869	0.865	0.136	0.088	0.077	6.090	9.868	11.254
	PB2	0.834	0.854	0.868	0.977	0.702	0.643	0.853	1.217	1.349
	CAN1	0.867	0.900	0.888	0.597	0.417	0.374	1.452	**2.158**	2.377
	CAN2	0.864	0.860	0.857	0.112	0.081	0.072	7.713	10.604	11.837
	VST1	0.852	0.874	0.882	0.581	0.410	0.370	**1.466**	2.129	**2.386**
$\rho_1 = 0.5$	VST2	0.873	0.857	0.870	0.127	0.085	0.075	6.866	10.043	11.546
&	BB1	0.868	0.900	0.887	0.616	0.420	0.374	1.408	2.143	2.372
$\rho_2 = 0.1$	BB2	0.842	0.852	0.849	0.104	0.078	0.070	**8.061**	**10.866**	**12.076**
	PB1	0.831	0.865	0.885	0.675	0.439	0.389	1.231	1.973	2.277
	PB2	0.856	0.854	0.861	0.109	0.080	0.071	7.873	10.726	12.074
	CAN1	0.852	0.886	0.877	0.589	0.419	0.376	1.446	**2.116**	2.332
	CAN2	0.836	0.869	0.875	1.020	0.726	0.641	0.819	1.197	1.366
	VST1	0.841	0.873	0.866	0.573	0.413	0.371	**1.466**	2.115	**2.333**
$\rho_1 = 0.5$	VST2	0.832	0.875	0.883	1.152	0.769	0.669	0.722	1.138	1.321
&	BB1	0.853	0.880	0.874	0.606	0.421	0.377	1.408	2.092	2.321
$\rho_2 = 0.9$	BB2	0.813	0.858	0.870	0.953	0.701	0.624	**0.854**	**1.225**	1.395
	PB1	0.836	0.866	0.869	0.660	0.439	0.391	1.267	1.973	2.224
	PB2	0.817	0.868	0.885	0.990	0.715	0.631	0.825	1.215	**1.402**
	CAN1	0.855	0.878	0.873	1.112	0.756	0.666	**0.769**	1.161	1.311
	CAN2	0.842	0.873	0.869	0.112	0.081	0.073	7.488	10.767	11.967
	VST1	0.832	0.870	0.867	1.083	0.744	0.658	0.768	**1.170**	**1.318**
$\rho_1 = 0.9$	VST2	0.837	0.876	0.875	0.127	0.086	0.076	6.604	10.236	11.531
&	BB1	0.861	0.877	0.872	1.143	0.759	0.666	0.753	1.155	1.309
$\rho_2 = 0.1$	BB2	0.820	0.861	0.860	0.105	0.078	0.071	**7.805**	10.997	**12.175**
	PB1	0.819	0.842	0.869	1.249	0.797	0.692	0.656	1.057	1.255
	PB2	0.836	0.877	0.865	0.109	0.080	0.072	7.656	**11.030**	12.090
	CAN1	0.879	0.874	0.887	1.109	0.751	0.676	0.793	1.164	1.312
	CAN2	0.840	0.864	0.860	0.550	0.406	0.356	1.527	2.130	2.419
	VST1	0.871	0.876	0.870	1.079	0.740	0.667	**0.807**	**1.184**	1.304
$\rho_1 = 0.9$	VST2	0.857	0.862	0.875	0.620	0.428	0.371	1.382	2.012	2.357
&	BB1	0.883	0.883	0.888	1.147	0.754	0.675	0.770	1.171	**1.315**
$\rho_2 = 0.5$	BB2	0.819	0.856	0.852	0.513	0.391	0.346	**1.598**	**2.187**	2.462
	PB1	0.859	0.875	0.862	1.259	0.792	0.703	0.682	1.105	1.227
	PB2	0.840	0.865	0.867	0.533	0.398	0.351	1.576	2.173	**2.472**

Table-4 : Simulation results of coverage percentage, average lengths and relative coverage for 90% confidence intervals of queueing Network $M/H_4^{Pe}/1$ to $H_4^{Pe}/M/1$

Intensity Parameters	Estimation Approches	Coverage Percentages			Average Lengths			Relative Coverage		
		$n=10$	$n=20$	$n=25$	$n=10$	$n=20$	$n=25$	$n=10$	$n=20$	$n=25$
	CAN1	0.884	0.897	0.892	0.128	0.088	0.077	**6.916**	**10.246**	11.615
	CAN2	0.855	0.873	0.882	0.578	0.420	0.379	1.478	2.078	2.325
	VST1	0.846	0.869	0.891	0.126	0.087	0.076	6.727	10.007	**11.678**
$\rho_1 = 0.1$	VST2	0.866	0.872	0.886	0.648	0.441	0.394	1.336	1.975	2.248
&	BB1	0.886	0.899	0.891	0.131	0.088	0.077	6.773	10.233	11.597
$\rho_1 = 0.5$	BB2	0.836	0.861	0.877	0.543	0.407	0.370	**1.539**	**2.117**	**2.370**
	PB1	0.833	0.860	0.886	0.141	0.092	0.080	5.892	9.375	11.113
	PB2	0.866	0.865	0.878	0.568	0.414	0.375	1.525	2.088	2.343
	CAN1	0.870	0.881	0.869	0.126	0.086	0.078	**6.919**	10.205	**11.198**
	CAN2	0.852	0.854	0.870	1.063	0.744	0.676	0.802	1.148	1.286
	VST1	0.848	0.879	0.860	0.124	0.086	0.077	6.853	**10.252**	11.162
$\rho_1 = 0.1$	VST2	0.853	0.854	0.876	1.201	0.782	0.703	0.710	1.092	1.245
&	BB1	0.868	0.883	0.867	0.130	0.087	0.078	6.692	10.172	11.159
$\rho_1 = 0.9$	BB2	0.831	0.847	0.862	0.999	0.721	0.660	**0.832**	**1.174**	**1.306**
	PB1	0.841	0.876	0.850	0.141	0.090	0.080	5.982	9.682	10.575
	PB2	0.842	0.850	0.871	1.041	0.735	0.669	0.809	1.156	1.302
	CAN1	0.871	0.890	0.893	0.640	0.437	0.386	**1.360**	**2.036**	**2.316**
	CAN2	0.845	0.875	0.867	0.118	0.085	0.075	7.154	10.310	11.537
	VST1	0.847	0.874	0.886	0.630	0.433	0.383	1.344	2.019	**2.316**
$\rho_1 = 0.5$	VST2	0.846	0.864	0.877	0.133	0.089	0.078	6.338	9.676	11.210
&	BB1	0.882	0.890	0.891	0.658	0.439	0.386	1.340	2.027	2.305
$\rho_1 = 0.1$	BB2	0.820	0.856	0.854	0.111	0.082	0.073	**7.381**	**10.418**	11.649
	PB1	0.832	0.867	0.894	0.720	0.458	0.400	1.156	1.892	2.233
	PB2	0.838	0.858	0.876	0.116	0.084	0.074	7.244	10.241	**11.787**
	CAN1	0.857	0.886	0.890	0.641	0.436	0.389	**1.337**	**2.034**	**2.287**
	CAN2	0.811	0.871	0.858	1.028	0.752	0.681	0.789	1.159	1.261
	VST1	0.825	0.876	0.883	0.632	0.431	0.387	1.306	2.031	2.284
$\rho_1 = 0.5$	VST2	0.830	0.875	0.859	1.148	0.790	0.708	0.723	1.108	1.214
&	BB1	0.857	0.886	0.891	0.666	0.437	0.391	1.287	2.027	2.279
$\rho_1 = 0.9$	BB2	0.788	0.861	0.851	0.967	0.729	0.663	0.815	**1.181**	**1.283**
	PB1	0.813	0.865	0.885	0.719	0.457	0.404	1.130	1.894	2.191
	PB2	0.822	0.873	0.854	1.007	0.742	0.674	**0.817**	1.176	1.268
	CAN1	0.861	0.882	0.889	1.151	0.780	0.700	**0.748**	**1.131**	1.269
	CAN2	0.829	0.889	0.884	0.116	0.083	0.074	7.131	10.650	11.891
	VST1	0.835	0.865	0.885	1.133	0.775	0.696	0.737	1.116	**1.272**
$\rho_1 = 0.9$	VST2	0.847	0.877	0.886	0.130	0.088	0.077	6.492	9.969	11.458
&	BB1	0.861	0.881	0.889	1.186	0.785	0.700	0.726	1.123	1.269
$\rho_1 = 0.1$	BB2	0.814	0.877	0.874	0.109	0.081	0.073	**7.462**	**10.841**	**12.054**
	PB1	0.834	0.862	0.876	1.292	0.820	0.727	0.646	1.051	1.205
	PB2	0.832	0.872	0.878	0.114	0.082	0.074	7.294	10.578	11.942
	CAN1	0.872	0.886	0.889	1.128	0.789	0.702	**0.773**	1.123	**1.267**
	CAN2	0.868	0.863	0.874	0.591	0.426	0.383	1.470	2.024	2.285
	VST1	0.848	0.882	0.870	1.111	0.783	0.695	0.763	**1.126**	1.251
$\rho_1 = 0.9$	VST2	0.853	0.871	0.891	0.664	0.450	0.398	1.285	1.936	2.237
&	BB1	0.868	0.884	0.887	1.159	0.796	0.703	0.749	1.111	1.261
$\rho_1 = 0.5$	BB2	0.847	0.854	0.862	0.555	0.414	0.373	**1.526**	**2.063**	2.314
	PB1	0.825	0.879	0.862	1.272	0.829	0.728	0.649	1.061	1.185
	PB2	0.858	0.860	0.883	0.579	0.421	0.379	1.482	2.041	**2.332**

Table-5 : Simulation results of coverage percentage, average lengths and relative coverage for 90% confidence intervals of queueing Network $E_4/H_4^{Pe}/1$ to $H_4^{Pe}/E_4/1$

Intensity Parameters	Estimation Approches	Coverage Percentages			Average Lengths			Relative Coverage		
		$n=10$	$n=20$	$n=25$	$n=10$	$n=20$	$n=25$	$n=10$	$n=20$	$n=25$
$\rho_1 = 0.1$ & $\rho_1 = 0.5$	CAN1	0.853	0.878	0.890	0.080	0.057	0.051	10.605	15.531	17.598
	CAN2	0.865	0.881	0.900	0.407	0.284	0.252	2.124	3.107	3.570
	VST1	0.843	0.885	0.889	0.080	0.056	0.050	10.506	15.709	17.625
	VST2	0.826	0.860	0.899	0.399	0.280	0.249	2.071	3.070	3.608
	BB1	0.830	0.866	0.880	0.075	0.055	0.049	**11.032**	**15.859**	**17.924**
	BB2	0.846	0.866	0.897	0.383	0.274	0.245	**2.207**	**3.157**	**3.664**
	PB1	0.836	0.878	0.892	0.079	0.056	0.050	10.624	15.758	17.833
	PB2	0.839	0.870	0.896	0.404	0.281	0.250	2.077	3.094	3.585
$\rho_1 = 0.1$ & $\rho_1 = 0.9$	CAN1	0.888	0.876	0.888	0.081	0.057	0.051	10.997	15.345	17.530
	CAN2	0.872	0.887	0.890	0.730	0.513	0.460	1.195	1.729	1.934
	VST1	0.867	0.866	0.883	0.081	0.057	0.050	10.745	15.222	17.498
	VST2	0.856	0.884	0.878	0.715	0.508	0.457	1.198	1.740	1.923
	BB1	0.867	0.860	0.875	0.075	0.055	0.049	**11.504**	**15.602**	**17.808**
	BB2	0.854	0.874	0.881	0.688	0.495	0.447	**1.241**	**1.767**	**1.972**
	PB1	0.874	0.865	0.882	0.079	0.056	0.050	11.085	15.348	17.635
	PB2	0.858	0.884	0.871	0.726	0.510	0.457	1.181	1.733	1.905
$\rho_1 = 0.5$ & $\rho_1 = 0.1$	CAN1	0.871	0.871	0.887	0.399	0.282	0.255	2.184	3.093	3.484
	CAN2	0.890	0.897	0.879	0.080	0.057	0.051	11.137	15.761	17.198
	VST1	0.850	0.876	0.888	0.397	0.280	0.254	2.140	3.127	3.501
	VST2	0.872	0.882	0.863	0.078	0.056	0.051	11.157	15.642	17.077
	BB1	0.854	0.855	0.874	0.373	0.272	0.247	**2.291**	**3.143**	**3.538**
	BB2	0.863	0.888	0.866	0.075	0.055	0.050	**11.478**	**16.170**	**17.448**
	PB1	0.863	0.871	0.887	0.389	0.277	0.251	2.217	3.139	3.529
	PB2	0.877	0.880	0.867	0.079	0.057	0.051	11.070	15.548	17.103
$\rho_1 = 0.5$ & $\rho_1 = 0.9$	CAN1	0.879	0.887	0.884	0.403	0.284	0.252	2.180	3.121	3.503
	CAN2	0.880	0.883	0.882	0.726	0.509	0.458	1.212	1.734	1.927
	VST1	0.871	0.874	0.880	0.401	0.283	0.251	2.170	3.087	3.503
	VST2	0.865	0.879	0.875	0.712	0.503	0.453	1.215	1.746	1.930
	BB1	0.859	0.872	0.873	0.376	0.274	0.245	**2.283**	**3.179**	**3.566**
	BB2	0.864	0.870	0.872	0.682	0.493	0.445	**1.267**	**1.765**	**1.961**
	PB1	0.871	0.882	0.879	0.394	0.280	0.249	2.213	3.149	3.530
	PB2	0.865	0.885	0.876	0.720	0.506	0.454	1.201	1.749	1.928
$\rho_1 = 0.9$ & $\rho_1 = 0.1$	CAN1	0.858	0.891	0.872	0.711	0.510	0.458	1.207	1.747	1.905
	CAN2	0.884	0.879	0.884	0.079	0.056	0.051	11.168	15.658	17.250
	VST1	0.854	0.884	0.865	0.707	0.508	0.457	1.207	1.740	1.893
	VST2	0.869	0.863	0.877	0.078	0.056	0.051	11.196	15.531	17.248
	BB1	0.839	0.877	0.864	0.665	0.493	0.445	**1.262**	**1.779**	**1.943**
	BB2	0.863	0.867	0.879	0.074	0.054	0.050	**11.601**	**15.978**	**17.653**
	PB1	0.856	0.878	0.869	0.694	0.503	0.453	1.233	1.746	1.920
	PB2	0.864	0.870	0.883	0.078	0.056	0.051	11.010	15.602	17.311
$\rho_1 = 0.9$ & $\rho_1 = 0.5$	CAN1	0.858	0.894	0.875	0.717	0.513	0.457	1.197	1.742	1.913
	CAN2	0.847	0.894	0.889	0.403	0.286	0.256	2.104	3.128	3.467
	VST1	0.838	0.888	0.876	0.713	0.512	0.456	1.175	1.735	1.920
	VST2	0.829	0.888	0.885	0.395	0.283	0.254	2.097	3.138	3.487
	BB1	0.835	0.881	0.860	0.668	0.495	0.444	**1.250**	**1.781**	**1.936**
	BB2	0.825	0.885	0.876	0.378	0.276	0.249	**2.181**	**3.208**	**3.516**
	PB1	0.838	0.891	0.873	0.699	0.506	0.452	1.199	1.761	1.931
	PB2	0.827	0.887	0.885	0.400	0.284	0.255	2.068	3.124	3.473

Table-6 : Simulation results of coverage percentage, average lengths and relative coverage for 90% confidence intervals of queueing Network $E_4/H_4^{Po}/1$ to $H_4^{Po}/E_4/1$

Intensity Parameters	Estimation Approches	Coverage Percentages			Average Lengths			Relative Coverage		
		$n=10$	$n=20$	$n=25$	$n=10$	$n=20$	$n=25$	$n=10$	$n=20$	$n=25$
$\rho_1 = 0.1$ & $\rho_1 = 0.5$	CAN1	0.881	0.882	0.885	0.079	0.057	0.051	11.120	15.505	17.401
	CAN2	0.864	0.868	0.898	0.397	0.283	0.253	2.174	3.063	3.549
	VST1	0.872	0.881	0.873	0.079	0.057	0.051	11.037	15.557	17.232
	VST2	0.852	0.863	0.891	0.390	0.280	0.251	2.185	3.078	3.551
	BB1	0.858	0.871	0.868	0.074	0.055	0.049	**11.590**	**15.867**	**17.596**
	BB2	0.845	0.858	0.890	0.374	0.274	0.246	**2.258**	**3.135**	**3.616**
	PB1	0.874	0.880	0.875	0.077	0.056	0.050	11.289	15.698	17.423
	PB2	0.851	0.857	0.884	0.394	0.281	0.251	2.159	3.050	3.516
$\rho_1 = 0.1$ & $\rho_1 = 0.9$	CAN1	0.871	0.882	0.884	0.081	0.057	0.051	10.762	15.445	17.311
	CAN2	0.876	0.886	0.911	0.707	0.515	0.454	1.240	1.720	2.004
	VST1	0.846	0.858	0.872	0.081	0.057	0.051	10.489	15.098	17.141
	VST2	0.864	0.880	0.900	0.692	0.509	0.450	1.248	1.729	1.998
	BB1	0.842	0.862	0.872	0.076	0.055	0.050	**11.120**	**15.663**	**17.570**
	BB2	0.847	0.872	0.903	0.665	0.497	0.441	**1.273**	**1.755**	**2.047**
	PB1	0.856	0.867	0.875	0.079	0.056	0.050	10.822	15.421	17.365
	PB2	0.873	0.881	0.900	0.700	0.510	0.452	1.247	1.726	1.992
$\rho_1 = 0.5$ & $\rho_1 = 0.1$	CAN1	0.864	0.869	0.909	0.399	0.284	0.257	2.163	3.057	3.541
	CAN2	0.874	0.875	0.882	0.079	0.057	0.051	11.043	15.273	17.334
	VST1	0.846	0.864	0.900	0.397	0.283	0.256	2.128	3.053	3.521
	VST2	0.855	0.873	0.877	0.078	0.057	0.050	11.008	15.429	17.411
	BB1	0.836	0.859	0.898	0.373	0.275	0.249	**2.239**	**3.129**	**3.603**
	BB2	0.856	0.861	0.874	0.074	0.055	0.049	**11.492**	**15.567**	**17.692**
	PB1	0.842	0.862	0.897	0.390	0.280	0.253	2.160	3.080	3.543
	PB2	0.858	0.864	0.870	0.079	0.057	0.051	10.895	15.197	17.222
$\rho_1 = 0.5$ & $\rho_1 = 0.9$	CAN1	0.874	0.891	0.887	0.397	0.281	0.254	2.200	3.169	3.486
	CAN2	0.878	0.880	0.880	0.722	0.513	0.457	1.216	1.717	1.924
	VST1	0.855	0.879	0.882	0.396	0.280	0.254	2.161	3.142	3.472
	VST2	0.869	0.853	0.870	0.707	0.506	0.452	1.230	1.685	1.923
	BB1	0.851	0.880	0.875	0.372	0.272	0.247	**2.287**	**3.241**	**3.538**
	BB2	0.860	0.863	0.873	0.679	0.495	0.445	**1.267**	**1.742**	**1.964**
	PB1	0.848	0.883	0.875	0.388	0.277	0.252	2.185	3.188	3.478
	PB2	0.864	0.854	0.874	0.716	0.508	0.454	1.206	1.680	1.924
$\rho_1 = 0.9$ & $\rho_1 = 0.1$	CAN1	0.857	0.890	0.869	0.712	0.517	0.458	1.203	1.721	1.897
	CAN2	0.874	0.881	0.880	0.081	0.057	0.051	10.803	15.503	17.302
	VST1	0.847	0.868	0.861	0.708	0.515	0.457	1.196	1.685	1.885
	VST2	0.840	0.867	0.886	0.079	0.056	0.050	10.617	15.426	17.580
	BB1	0.838	0.880	0.865	0.666	0.499	0.445	**1.259**	**1.764**	**1.945**
	BB2	0.854	0.868	0.874	0.076	0.055	0.049	**11.223**	**15.844**	**17.692**
	PB1	0.851	0.870	0.867	0.694	0.509	0.453	1.225	1.708	1.913
	PB2	0.833	0.870	0.883	0.080	0.056	0.051	10.389	15.422	17.480
$\rho_1 = 0.9$ & $\rho_1 = 0.5$	CAN1	0.865	0.889	0.904	0.709	0.509	0.457	1.220	1.745	1.979
	CAN2	0.865	0.882	0.895	0.401	0.286	0.252	2.158	3.089	3.552
	VST1	0.842	0.880	0.891	0.706	0.507	0.455	1.193	1.734	1.958
	VST2	0.843	0.878	0.897	0.393	0.282	0.249	2.147	3.109	3.596
	BB1	0.838	0.880	0.893	0.663	0.492	0.444	**1.265**	**1.790**	**2.013**
	BB2	0.834	0.873	0.888	0.377	0.276	0.245	**2.213**	**3.165**	**3.629**
	PB1	0.848	0.886	0.894	0.691	0.502	0.451	1.227	1.764	1.981
	PB2	0.840	0.880	0.894	0.397	0.284	0.250	2.114	3.104	3.574

Note that:

1. boldface denotes the greatest relative coverage among estimation approaches.
2. confidence intervals for ρ_1 under different estimation approaches are denoted by CAN1, VST1, BB1, PB1 and that of ρ_2 are denoted by CAN2 , VST2, BB2, PB2.

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