STATISTICS, OPTIMIZATION AND INFORMATION COMPUTING
Stat., Optim. Inf. Comput., Vol. **12**, May 2024, pp 829–840.
Published online in International Academic Press (www.IAPress.org)

Optimal reconstruction and recognition of images by Jacobi Fourier moments and artificial bee Colony (ABC) algorithm

Yahya Sahmoudi [1,*], Jaouad El-Mekkaoui [1], Boujamaa Janati Idrissi[1], Omar El Ogri [1;2], Mohamed Benslimane [1], Amal Hjouji [2] , Karim El Moutaouakil [3]

[1] *LTI, Innovative Technologies Laboratory, Higher school of Technology, Sidi Mohamed Ben Abdellah University, Fez, Morocco*
[2] *CED-ST, STIC, Laboratory of Information, Signals, Automation and Cognitive LISAC, Sidi Mohamed Ben Abdellah University, Fez, Morocco*
[3] *Engineering Science Laboratory, Polydisciplinary Faculty of Taza, Sidi Mohamed Ben Abdellah University of Fez, Morocco*

Abstract The orthogonal moments giving relevant results of these last years within the framework of object detection, pattern recognition and image reconstruction, this work based on orthogonal functions called Orthogonal Jacobi Polynomials (OJPs), and we introduce a new set of moments called Generalized Jacobi Fourier Moments (GJFMs), these polynomials are characterized by parameters α,β and λ. However, it was very important to optimize these parameters in order to obtain a good result, in this context; this study used a new approach to optimized Jacobi Fourier parameters α,β and λ using the artificial bee colony algorithm (ABC) in order to improves the quality of reconstruction of images of large sizes. On the one hand, to validate this technique which offers a high image reconstruction quality. On other hand, the comparison carried out with other algorithms clearly indicates the advantage of the proposed method.

Keywords Orthogonal Jacobi polynomials, Generalized Jacobi Fourier moments, images reconstruction, Artificial bee Colony (ABC).

DOI: 10.19139/soic-2310-5070-1973

1. Introduction

For image recognition and representation images one applies one of the techniques which transform an image into a vector by requiring a decision to be made based on the class specific to that image. clearly, the extracted feature vector which gives the quality of the image representation. For this, in recent years, applications on image analysis and pattern recognition known very important developments. The importance of orthogonal moments (OMs) in geometric transformations which are introduced by Teague [1] in 1980, Orthogonal moments are more powerful when representing im-ages with low information redundancy and good noise resistance [2], than non-orthogonal moments, and in variety of image processing applications, including, reconstruction [3], classification [4], resolution [5][6], image detection [7], video encoding [8], image representation [9][13] and compression [14].

The Jacobi Fourier moments (JFMs) introduced by Ping et al. [15] are orthogonal moments defined by multiplying two radial and angular kernel functions, shift function and other angular Fourier function, this Jacobi polynomials depending on three control parameters α , β and Λ. Hoang and Tabone [16] take into consideration the points presented in the definition of the radial kernels as defined in [15]. Zernik moments (ZMs) [17], pseudo Zernik moments (PZMs) [18], Mellin Fourier moments (MFMs) [19], Chebyshev Fourier moments (CHFMs) [20]

*Correspondence to: Yahya Sahmoudi (Email: yahya.sahmoudi@usmba.ac.ma). LTI, Innovative Technologies Laboratory, Higher school of Technology, Sidi Mohamed Ben Abdellah University, Fez, Morocco.

ISSN 2310-5070 (online) ISSN 2311-004X (print)

pseudo Jacobi Fourier moments (PJFMs) [21], and Legendre Fourier moments (LFMs) [22] are some particular examples of JFMs based on control parameters [15].

In this article, we present an optimal and coherent method for optimizing Jacobi Fourier parameters α , β and Λ. To evaluate these applications concerning the quality of the reconstructed images, we have to use the mean square error (MSE). For a better quality of reconstructed images, we must find optimal values of these Local parameters (α , β and Λ), the problems of the selection of the parameters of the functions depend on the processing situation that we need, in our work, we propose a powerful method for selection of the parameters of the Orthogonal Jacobi Polynomials (OJPs), we rely on the optimization instruction method called Artificial Bee Colony (ABC) [23]. the latter is used to select the good values of the OJPs function parameters which give a better reconstruction of the images thanks to the minimization of the MSE.

The performance of the ABC algorithm for the optimization of the parameters of the OJPs functions provides the motivation to present an optimal method for the reconstruction of large images based on generalized Jacobi Fourier moments (GJFMs) and ABC algorithm. However, the rest of this work is organized as follows: In the 2nd section, we present generalities on the calculation of OJPs. In the 3rd section, we present the accurate computational method of the moments GJFMs. The 4th section the optimal reconstruction of images by moments GJFMs using the ABC algorithm are presented. In the 5th section, we present the proposed model's architecture based on the support vector machines (SVM) [24] and moments GJFMs optimized for classification. The robustness of the proposed reconstruction method in the last section.

2. Generalized orthogonal Jacobi Fourier moments

The Generalized orthogonal Jacobi Fourier moments GJFMs are defined as follows:

$$GJFM_{pq}^{(\alpha,\beta,\lambda)} = \frac{1}{2\pi A_p(\alpha,\beta,\lambda)} \int_0^{2\pi} \int_0^1 f(r,\theta) \cdot J_p(\alpha,\beta,\lambda,r) \cdot e^{-iq\theta} \cdot B(\alpha,\beta,\lambda,r) \cdot r dr d\theta \tag{1}$$

Where p and q are the order and repetition; $p = |q| = 0; 1; 2; \ldots \infty$

The normalization constant $A_p(\alpha,\beta,\lambda)$, the weight function $B(\alpha,\beta,\lambda,r)$ and the Jacobi polynomials $J_p(\alpha,\beta,\lambda,r)$ are defined as follows:

$$A_p(\alpha,\beta,\lambda) = \frac{p!((\beta-1)!)^2(\alpha-\beta+p)!}{\lambda(\alpha-1+p)!(\beta-1+p)!(\alpha+2p)!} \tag{2}$$

$$B(\alpha,\beta,\lambda,r) = \left(1-r^{\lambda}\right)^{\alpha-\beta}\left(r^{\lambda}\right)^{\beta-1}; \alpha-\beta > -1 \text{ and } \beta-1 > -1 \tag{3}$$

$$J_p(\alpha,\beta,\lambda,r) = \frac{p!(\beta-1)!}{(\alpha+\beta-1)!}\sum_{k=0}^{p}(-1)^k \frac{(\alpha+\beta+k-1)!r^{\lambda k}}{(p-k)!(k)!(\beta+k-1)!}; \lambda \in \mathbb{R}^+ \tag{4}$$

The radial functions $J_p(\alpha,\beta,\lambda,r)$ are orthogonal in the interval $0 \leq r \leq 1$ and the Fourier exponential $e^{-iq\theta}$ as angular function. The image function $f(r,\theta)$ could be reconstructed as follows:

$$f(r,\theta) = \sum_{p=0}^{\infty}\sum_{q=-\infty}^{\infty} GJFM_{pq} J_p(\alpha,\beta,\lambda,r)e^{iq\theta} \tag{5}$$

With pre-defined, $p_{\max}$ and $q_{\max}$ equation (5) becomes:

$$\hat{f}(r,\theta) \sim \sum_{p=0}^{p_{\max}}\sum_{q=-q_{\max}}^{q_{\max}} GJFM_{pq} J_p(\alpha,\beta,\lambda,r)e^{iq\theta} \tag{6}$$

3. Accurate Computational Method of the GJFMs

The Generalized orthogonal Jacobi Fourier moments GJFMs of order pq of any function $f(r,\theta)$ defined on $\theta \in [0,2\pi]$ and $r \in [0,1]$. We must adopt the cartesian image pixels to the polar image pixels technique to calculate the moments GJFMs on images of size $2N \times 2N$ with concentric circles. We use the ri direction to divide the disk into N rings, we limit each ring by two circles of $\left\{r_i = \frac{\mathrm{i}}{\mathrm{N}} \text{ and } r_{i+1} = \frac{\mathrm{i+1}}{\mathrm{N}}; i = 0,1,2\ldots,N-1\right\}$, and each number i of ring contains num $4+8i$ equals parts determined by the angles θ_{ix}.

Therefore, we calculate the $GJFM_{pq}$ moments using the zeroth-order approximation mer (ZOA) [25] for a digital image intensity function $f(r,\theta)$ of size $2N \times 2N$:

$$GJFM_{pq}^{(\alpha,\beta,\lambda)} = \frac{1}{2\pi A_p(\alpha,\beta,\lambda)} \sum_{i=0}^{N} \sum_{x=0}^{3+8i} f\left(r_i,\theta_{ix}\right) J_p\left(\alpha,\beta,\lambda,r_i\right) e^{-jq\theta_{ix}} \nabla r_i \nabla \theta_{ix} \tag{7}$$

$J_p\left(\alpha,\beta,\lambda,r_i\right)$ are the pth orthogonal polynomials of Jacobi

$$\theta_{ix} = \frac{2\left(x+\frac{1}{2}\right)\pi}{4+8i}, x = 0,\ldots,3+8i \tag{8}$$

$$\nabla r_i = r_{i+1} - r_i = \frac{i+1}{N} - \frac{i}{N} = \frac{1}{N} \tag{9}$$

$$\nabla \theta_{ix} = \theta_{i,x+1} - \theta_{i,x} = \frac{2\pi(x+1+0.5)}{4+8i} - \frac{2\pi(x+0.5)}{4+8i} = \frac{2\pi}{4+8i} \tag{10}$$

An approximation $\tilde{f}(r,\theta)$ of the original image can be reconstructed by:

$$\tilde{f}(r,\theta) = \sum_{p=0}^{p_{\max}} \sum_{q=-m_{\max}}^{q_{\max}} GJFM_{pq}^{(\alpha,\beta,\lambda)} J_p(\alpha,\beta,\lambda,r) e^{jq\theta} \tag{11}$$

From the processing we have done we can say that this approach is limited by two problems: the quality of the processed images influenced by approximation errors and large orders requiring very high calculation time. to solve these problems, we calculate 2D GJFMs moments in two cascaded steps by successive calculation of the corresponding 1D GJFMs for each line. This precise and rapid method for calculating generalized Jacobi Fourier moments.

4. Optimal reconstruction of images by GJFMs and the ABC algorithm

The optimal reconstruction of images by GJFMs using the ABC algorithm are presented in Figure 2.

ABC optimization algorithm for image reconstruction is based on the following steps:

The first step: The parameters of the algorithm ABC will be initialized

The initial values used in our algorithm are:

Number of variables, $N = 2$

The number of iterations, $R = 20$

The food source size, $F = 4$

The repeat condition, limit $= (\boldsymbol{F} \times \boldsymbol{N} \times \boldsymbol{N}) \div 2$

The condition of parameters: between $S\min = 0$ and $S\max = 100$:

The 2nd step: The initial population must be generated randomly, using the following equation (12)

$$F_i = S_{\min} + \operatorname{rand}(0,1) \times \left(S_{\max} - S_{\min}\right) \tag{12}$$

Where F_i the solution found, the space of games made between Smin, Smax (the range of r_x and r_y) and rand $(0,1)$ random function of continuous distribution on $[0;1]$.

Using the MSE function (13) to evaluate each generated solution F_i

$$MSE=\frac{1}{(\max+1)\,(\max'+1)}\sum_{p=0}^{\max}\sum_{q=0}^{\max}(f(r,\theta)-\tilde{f}(r,\theta))^2 \tag{13}$$

The values that must be modified for each iteration are organized as follows;

The solution found F_i	$r_{x,i}$	$r_{y,i}$	MSE_i	$T_i=0$
F_1	$r_{x,1}$	$r_{y,1}$	MSE_1	$T_i=0$
F_2	$r_{x,2}$	$r_{y,2}$	MSE_1	$T_i=0$
....	....	....	....	....

Figure 1. The organization of initial variables

Trace (T_i) in the first step takes the value 0 , this is the number of iterations which does not minimize the MSE_i
The 3nd step: Employed bees phase, six instructions in this step
We will improve the values of the parameters r_x and r_y in all F_i
$i=0$
We choose a random number from among $r_{x,i}$ Or $r_{y,i}$ in F_i : Either the value is $r_{x,i}$
We choose a partner number $r_{x,n}$ from among F_n with $n\neq i$
New solution generated using the following equation:

$$r_{x,iNew}=r_{x,i}+\mathrm{rand}(0,1)\times(r_{x,i}-r_{x,n}) \tag{14}$$

We calculate MSE_{ivew} by new parameter $r_{xi\text{ ivew}}$, and r_{yi}
We compare MSE_{ivew} with MSE_i

$$\text{if } MSE_{\text{iNew}} < MSE_i \text{ then}$$

$$MSE_i=MSE_{iNew}$$

$$r_{xii_i}=r_{x,\text{ iNew}}$$

$$\text{else}$$

$$T_i=T_i+1$$

$$t=i+1$$

Step 3 must be repeated

***The 4nd step**:* Onlooker bees phase
Using the following equation to calculate the probability value of each solution MSE,

$$P_i=\frac{MSE_i}{\sum_1^F MSE_i} \tag{15}$$

$$Letr=\mathrm{rand}(0,0.5)$$

$$\text{if } P_{\text{i}} > r \text{ then}$$

Step 3 must be repeated

$else$

Step 4 must be repeated

The 5nd step: Scout bees phase
if $T_i >$ limit
then

using Eq. 12

end
The 6nd step: Steps 3,4 and 5 repeated, we stop at the R (Repetition numbers)
The 7nd step: Recording of the parameters r_x and r_y obtained and the image reconstructed in order n.

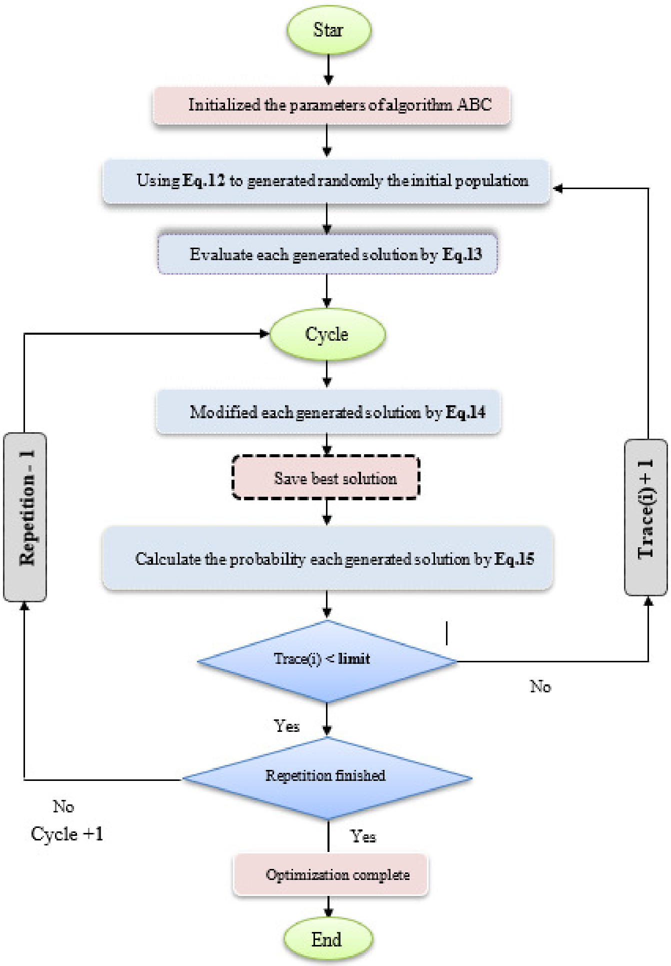

Figure 2. Bee Colony (ABC) Algorithm Flowchart

5. The proposed model's architecture

After pre-processing, we obtain a characteristic vector which allows SVM to do the learning. Figure 3 clearly clarifies the proposed method. To do the test, we take a test image, we calculate the GLPMIs moments to obtain the final characteristic vector for the test image, so to get the decision about the target value we compare with the data space of the SVM decision limit.

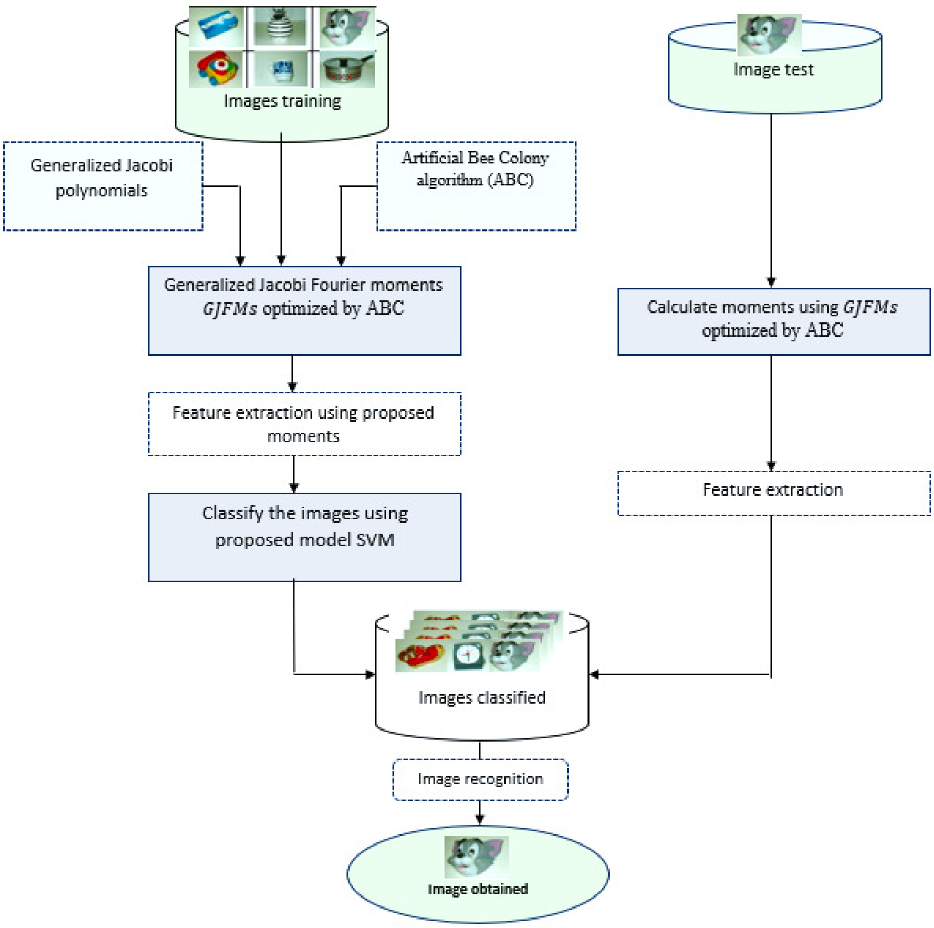

Figure 3. Classification method process

6. Experimental results and discussions

The performance and efficiency of the proposed optimization technique for image reconstruc-tion is tested by the results obtained in this section. This experiment contains two parts: The 1st test part we show the power of the ABC algorithm with GJFMs in the image color recon-structed. In the 2nd part we offer the recognition Rates using GJFMs.

6.1. Images reconstructed by ABC algorithm and GJFMs moments

Using two color images: Figue 4 of sizes 128 × 128 to test the proposed method for image reconstruction. Using the ABC algorithm optimization for GJFMs moments for orders from 10 to 63 concerning the reconstructed images. We present in Figure 5 the images reconstructed in the different orders by the GJFM moments which are optimized by the ABC algorithm and also present the values of local parameters α, β and λ , we observe that the reconstructed image quality increases proportionally with the order moments GJFMs, and the values of the local parameters do not follow any rule which shows the strength of the selection of parameters by the ABC algorithm, we see that the proposed method is reliable for the optimal reconstruction of color images.

We must evaluate the quality of the proposed method on color images by two tests, 1st assessment is compared with the classical method without optimizing the polynomial parameters while 2nd assessment the ABC algorithm

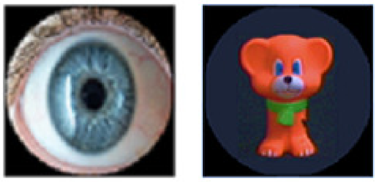

Figure 4. Originals images (a) and (b)

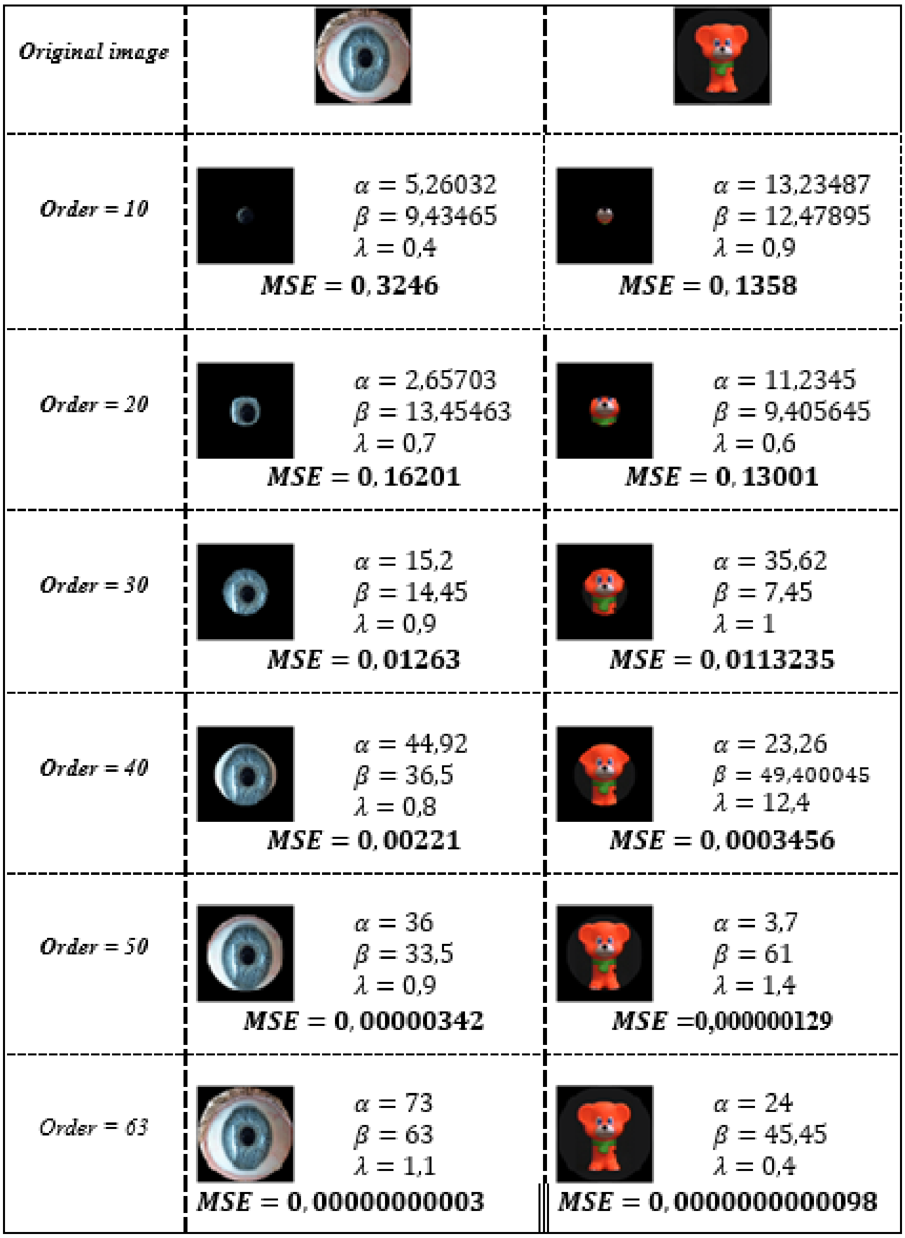

Figure 5. Reconstructed color images ‘ 128×128 ‘ using the proposed moments GJFMs optimized by ABC

has been compared by some algorithms of optimization; Fir fly Algorithm (FA)[26], Ant Colony Optimization (ACO)[27], Differential Evolution (DE) [28], Learning Teaching Based Optimization (TLBO)[29], and Particle Swarm Optimization (PSO)[30],

1st test, the proposed method based on GJFMs moments optimized by ABC algorithm is compared by the classical method which is based only on GJFMs moments. From Figure 6, we notice that the MSE curves for image reconstruction “Figure 4 (a)” decreases when the reconstruction order increases and approaches zero, which implies that the quality of the reconstruction increases when we increase the order value of moments GJFMs. Furthermore, it is clearly appreciated that the ABC algorithm allows to reach a minimal MSE compared to the conventional method, which justifies the robustness of the optimization method based on the ABC algorithm for image color reconstruction.

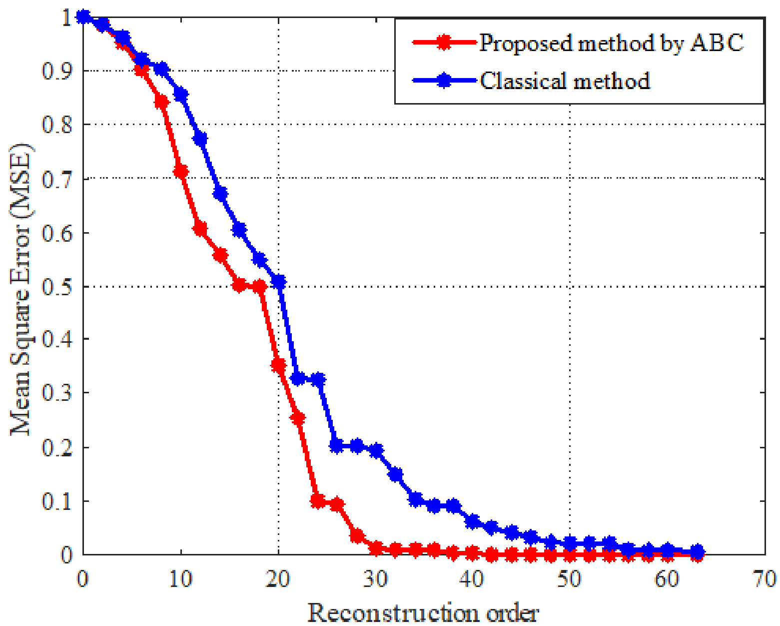

Figure 6. Reconstructed images errors MSE with different maximum order of moments used classical method and proposed method optimized

2nd test, we use another method to validate our GJFMs moments optimized by ABC algorithm proposed for the reconstruction, to do so we compare errors obtained by the ABC algorithm errors obtained by some optimization algorithms (ACO, PSO, DE and TLBO) during the reconstruction. Mean Square Error (MSE) of the ” Figure 4 (b) ” image reconstructed based on GJFMs optimized by ABC and other optimization algorithms are displayed in figure 7. Through this figure we can observe that the reconstruction errors based on our optimization method is much more optimal than that obtained by the other algorithms.

6.2. Recognition rates using GJFMs

In this subsection, we must use the technique to evaluate the effectiveness of proposed method GJFMs, is the GJFMs-SVM architecture used the SVM classifier. Two databases were used; the ETHZ-70Obj database [31] contains 270 objects classified in 70 categories where the image sizes are 320×240 and the COIL-100 database [32] contains 7202 color images classified in 100 categories with a size unified image of 128×128 to make several experiments and tests. We selected from the data illustrated in figures 8 and 9 at random examples of images to test our technique in a way to make several transformations, rotation, translation, scaling and mixed, in order to generate the objects from the COIL database and the objects from the ETHZ database, so we created two additional

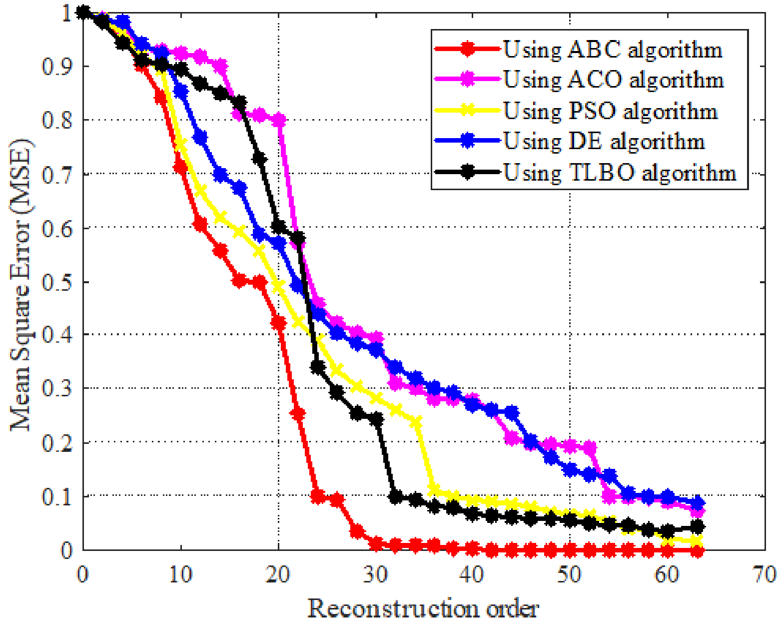

Figure 7. Reconstructed images errors MSE with different maximum order of moments used proposed method by ABC algorithm and the optimization algorithms: ACO, PSO, DE and TLBO

databases by adding different speckle noise densities, to see the performance of proposed GJFMs optimized by ABC algorithm moments about the accuracy of the classification, we compare the accuracy obtained during the classification using the GJFMs and SVM optimized by ABC algorithm with the other accuracy obtained by the optimization algorithms: ACO, PSO, DE and TLBO.

We presented in the tables 1,2, 3, 4, and 5 the following evaluation parameters: Noise-free, speckle noise accuracy for each database. From these tables, we can observe that the values found by adopting the ABC algorithm outperform those of the other algorithms. This seems clear when focusing on the value of overall precision: 99.02/100 and 91.60/100 when using ABC algorithm, 88.56/100 and 89.44/100 when using PSO algorithm, 87.51/100 and 88.27/100 when using TLBO algorithm, 86.34/100 and 87.05/100 when using DE algorithm and .86,49/100 and 84.89/100 when using ACO algorithm.

Figure 8. The selected images from database: ETHZ-70Obj

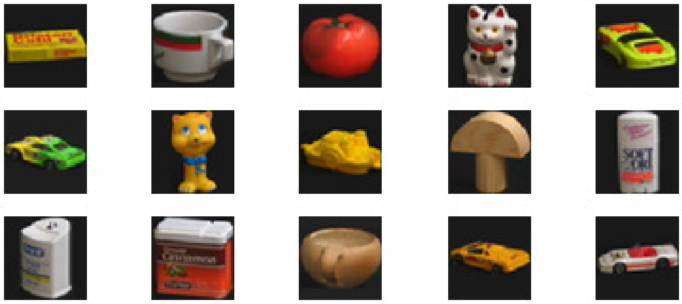

Figure 9. The selected images from database: COIL-100

Table 1. Object recognition accuracy on ETHZ-70Obj [31] and COIL-100 [32] databases, by using proposed technique GJFMs-SVM optimized by ABC algorithm.

Database	Noise free	Speckele noise					Average
		0.2%	0.4%	0.6%	0.8%	1%	
COIL-100 [32] database	99.89	96.13	92.12	89.60	88.13	86.27	**92.0233333**
ETHZ-70Obj [31] database	99.91	94.33	91.08	90.04	89.57	84.71	**91.6066667**

Table 2. Object recognition accuracy on ETHZ-70Obj [31] and COIL-100 [32] databases, by using proposed technique GJFMs-SVM optimized by PSO algorithm.

Database	Noise free	Speckele noise					Average
		0.2%	0.4%	0.6%	0.8%	1%	
COIL-100 [32] database	99.14	92.23	89.16	86.63	84.16	80.04	**88.56**
ETHZ-70Obj [31] database	98.94	93.13	90.10	86.85	85.30	82.32	**89.44**

Table 3. Object recognition accuracy on ETHZ-70Obj [31] and COIL-100 [32] databases, by using proposed technique GJFMs-SVM optimized by TLBO algorithm.

Database	Noise free	Speckele noise					Average
		0.2%	0.4%	0.6%	0.8%	1%	
COIL-100 [32] database	98.29	91.24	88.20	85.561	83.096	78.69	**87.512833**
ETHZ-70Obj [31] database	97.97	91.93	89.12	85.66	84.03	80.93	**88.273333**

Table 4. Object recognition accuracy on ETHZ-70Obj [31] and COIL-100 [32] databases, by using proposed technique GJFMs-SVM optimized by DE algorithm.

Database	Noise free	Speckele noise					Average
		0.2%	0.4%	0.6%	0.8%	1%	
COIL-100 [32] database	97.32	90.04	87.22	84.37	81.82	77.30	**86.345**
ETHZ-70Obj [31] database	97.12	90.94	88.16	86.59	82.96	76.58	**87.058333**

7. Discussions

The choice of local parameters of polynomials remains one of the necessary areas for development and research, for which we used in this work the ABC algorithm to resolution the local parameters of Orthogonal Jacobi Polynomials (OJPs). using OJPs polynomials to introduce a new group of moments called Generalized Jacobi-Fourier (GJFMs) which are used for the images reconstruction, to guarantee the choice of local optimization parameters (α,β and λ) we used the ABC optimization algorithm during image reconstruction. The simulation results show the power

Table 5. Object recognition accuracy on ETHZ-70Obj [31] and COIL-100 [32] databases, by using proposed technique GJFMs-SVM optimized by ACO algorithm.

Database	Noise free	Speckele noise					Average
		0.2%	0.4%	0.6%	0.8%	1%	
COIL-100 [32] database	98.12	91.90	88.10	85.12	80.16	75.57	**86.495**
ETHZ-70Obj [31] database	97.93	92.56	89.34	81.788	75.89	71.89	**84.8996667**

in terms of choice of local parameters (α, β and λ) of the proposed method compared to classical methods and in terms of quality of images reconstruction. In addition, the proposed method must be used in the biometric recognition, biomedical, implementation of innovated smart city applications, such as smart metering, smart farming, smart logistics, and smart buildings, hybridizing ABC with novel optimization techniques, such as Salp Swarm Algorithm (SSA) [33], Whale Optimization Algorithm (WOA) [34], Lion Optimization Algorithm (LOA) [35], Elephant Herding Optimization (EHO) [36],etc . . . fields, which also constitutes one of the axes of our future work.

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