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Optimization techniques of Assignment Problem using Trapezoidal Intuitionistic Fuzzy Numbers and Interval Arithmetic

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Abstract This paper discusses the Assignment problem to optimize the assigning of jobs to workers based on their talents and efficiency. In general, scheduling jobs plays a significant role in manufacturing and is advantageous in real world applications as we face more uncertainty and ambiguity in assigning jobs. The Intuitionistic Fuzzy Assignment problem (IFAP) is employed in circumstances when decision-makers have to deal with uncertainty. The domains are Trapezoidal Intuitionistic Fuzzy Numbers (TrIFNs) and the techniques used are Hungarian Method (HM), Brute Force Method (BFM), and Greedy Method (GM). The suggested model's performance is compared with the existing approach with the help of interval arithmetic operations. Allocating work to the individual is illustrated numerically, the optimal solution of minimizing cost is obtained using R programming and the results of comparative analysis are shown diagrammatically that help viewers to easily understand and generate results from comparisons.

Keywords Assignment problem, Trapezoidal Intuitionistic Fuzzy numbers, Interval Arithmetic, Hungarian method.

AMS 2010 subject classifications 90C08, 03E72

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1. Introduction

Fuzzy set (FS) is used in various fields such as image processing, Engineering disciplines, optimization etc. The idea of this theory is discovered by Zadeh [1] which is an useful and efficient instrument which can help with the problems related to judgements that are vague, unclear, and opaque. In this condition, the major fault is the performance of the idea of FS completely built upon human knowledge in which he/she may not be clear in the conclusion, since the value is connected to the accurate real number. In recent times, the values are inaccurate with more unpredictability and hence, Interval-valued Fuzzy set (IVFS) introduced by Zadeh comes to be a great tool for discussing optimization problems under uncertainty. Moreover, FS and IVFS is not suitable to all conditions due to the lack of complexity, uncertainty handling etc., and therefore, Atanassov [2] in 1983 introduced the Intuitionistic Fuzzy Set (IFS). IFS is a practical tool to deal with the problems involving vagueness and hesitation. IFS characterizes not only membership degree but also non-membership degree, examines the real life decision making problems. To make this more better, Atanassov extended IFS into Interval-valued Intuitionistic Fuzzy Set (IVIFS) [3] to treat the vagueness in which the values are intervals.

Assignment problem (AP) is derived from Linear programming problem, involves allotment of several resources to numerous actions and the only goal is to lower down the cost or time and to upraise the total profit or sale. AP is solved by developing a computational method called HM by Kuhn [4] in 1955 in a effective manner. This technique

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is frequently employed in many different disciplines, including operations research, logistics, transportation, and production planning, where optimal assignment is an essential factor. AP is beneficial in reaching a conclusion in regard to which job should be assigned to which employee, which class should be allotted to which teacher, etc. Due to uncertainty and vague information, it is hard to select a proper person for various job roles and it can be cleared up by AP. Thakre et al. [5] judged the placement of four candidates to four various selections in LIC and it is done by Fuzzy Assignment Problem (FAP), by transforming fuzzy values into crisp by magnitude ranking method and then resolved by HM, Matrix One's Assignment and Direct method. Sakthivel [33] suggested a novel ranking technique to clear jumbled data and solve FAP using the Hungarian Method. A numerical illustration is provided with TFN parameters. Thiruppathi et al. [6] utilized the centroid method in converting hexagonal FAP into crisp and found the optimal solution. Dhouib [35] introduced the AP with TrFN parameters and solved it using the innovative Dhouib-Matrix-AP1 (DM-AP1) heuristic, which consists of three phases with n iterations, and uses Python to construct an assignment network. In AP, the values of the cost are not be in crisp always, the parameters are unsure and those boundaries are described in interval by Ramesh and Ganesan [8] and proposed a computational approach to deal with the AP by using generalized Interval arithmetic Hungarian method. Ganesh et al. [7] introduced the process for solving interval data based FAP involving fuzzification, defuzzification and the solution is based on identifying right shape among different shapes. Ashwini et al. [9] adopted FAP and the cost is examined as triangular and trapezoidal number to optimize the cost using One's Assignment Method and Robust's ranking method.

IFAP is built since traditional fuzzy sets cannot handle imprecise or missing information. This flexibility is useful when dealing with unclear or unavailable real-world data. Lone et al. [10] used IFAP to provide minimum fertilizer cost to the farmer in which the accuracy function is utilised to defuzzify the cost into crisp and the optimal allotment of AP is formulated by Branch and Bound method. Senthil Kumar [34] formulated crisp, fuzzy, and intuitionistic fuzzy optimisation problems utilising a new theorem for type-2 and intuitionistic fuzzy optimisation. This study presents seven examples, solves FAP and IFAP with software, and compares outcomes using superiority analysis. Amutha et al. [11] investigated and obtained an optimal solution for IFAP by employing Intuitionistic Fuzzy numbers (IFNs) of symmetric Octogonal, also the cost are defuzzified into crisp using ranking and it is solved by HM. Dhouib [12] proposed the unique constructive heuristic DM-AP1 with a timing complexity of O(n) is used to solve AP using Triangular Intuitionistic Fuzzy Numbers (TIFNs). Prabha et al. [13] dealt with allocation and scheduling of the work for solving IFAP involving TrIFNs by three various methods viz., Intuitionistic Fuzzy Reduced Matrix method, Intuitionistic Fuzzy Ones assignment method and Intuitionistic Fuzzy Approximation method. Sahoo [14] proposed IFAP in regarding the increasing and decreasing of the market economy to represent the impreciseness, different approaches have used such as interval, stochastic and fuzzy stochastic and it is applied by ranking procedure of IFNs and HM in which the costs are TIFNs. To discuss with the ambiguousness even more, Interval valued Intuitionistic fuzzy Assignment problem (IVIFAP) is modelled by Traneva et al. [15] in which the parameters are taken as the Interval valued Intuitionistic fuzzy numbers (IVIFNs) to identify optimal assignment of jobs, on the basis of Index matrices and IVIFS is implemented without transforming into a traditional linear problem. Pothiraj et al. [16] suggested IVIFAP including the replacements using IVIFNs and the performance of a person depends on the time taken to do a particular work and it is shown numerically. An AP is a subset of the Transportation problem (TP) in which the number of sources and destinations is set to one. Sanjana et al. [17] solved TP with parameters as IVIFNs by using various techniques and interval arithmetic operations.

To avoid the uncertainty, Kar et al. [18] developed TrIFNs of type-2 to work out an AP using HM with the help of arithmetic Operations of Type 2 TrIFNs. Guneri et al. [19] emphasized the significance of heuristic solution for the classical AP using R program using various methods such as BFM, GM etc. Nagoorgani et al. [20] suggested an algorithm for generalized TrIFNs in the basis of labeling technique and the assignment matrix is defined by utilizing the ranking method to find the optimal solution of AP. Jansi Rani [32] demonstrated the significance of IVIFNs by solving a transportation problem utilising a novel subtraction operation and the diagonal optimal approach for Interval-valued Trapezoidal Intuitionistic Fuzzy Numbers (IVTrIFNs). Kodukalla [31] developed a new approach for ranking IVTrIFNs based on the centre of gravity of hesitation degree. The proposed method for solving AP is compared to existing methods using numerical examples to demonstrate its superiority. A numerical illustration is taken from Nagoorgani and Mohamed [21] developed AP with cost as Generalised Trapezoidal Intuitionistic numbers (GTrIFNs) by using the ranking technique and a new method is presented which are used for all types of AP. The major goal of this paper is to obtain optimal solution of IVIFAP with TrIFNs and various methods are used to schedule work to employee using interval arithmetic operations. Therefore, the primary innovation is that we can examine the non-membership value while using an IFS, and we can range the values by using intervals. The comparison is displayed to highlight how much more effective our solution is than the existing approach.

Objective: The IVIFAP aims to establish the most suitable job assignment for employees, that improves overall fulfilment in both employees and the company as a whole. The major characteristics to be carry out in solving IFAP are,

- The given TrIFNs are converted into interval numbers, which are a natural way of expressing uncertainty. Intervals allow us to describe a range of possible outcomes rather than crisp value when we have inaccurate or limited data.
- For allocating suitable job to the workers, the proposed approaches for solving AP in an intuitionistic fuzzy environment are HM, BFM, and GM.
- To give the most optimal results, the acquired intervals are subsequently changed to the midpoint and width using the interval and TrIFNs arithmetic operations.
- Finally, a comparison is done with the existing method, which produces the most optimal solution of the existing methods, as well as a schematic representation is provided to understand easily.

Purpose of the study: The paper's primary findings are,

- This study contributes that the intuitionistic fuzzy environment helps to interpret the ambiguous information when addressing the assignment problem and every element of the assignment matrix have TrIFNs which is the objective function that include degrees of membership and non-membership.
- The trapezoidal shape allows decision-makers to convey varying levels of certainty over each assignment. These numbers provide more precise and detailed depiction of the decision-making situation.
- Here, TrIFNs are converted into IVTrIFNs, and so IVTrIFNs enable decision-makers to indicate uncertainty. The main highlight of our study is that, handling uncertainty is not only in the degree of membership and non-membership, but also in the parameters of the trapezoidal membership function.
- The usage of TrIFNs in the IVIFAP is useful in real-world decision-making contexts involving uncertainty and imprecision. Real-life examples like project assignment, job allocation etc., are used for decision-makers when they are unsure about the value of specific assignments.
- In this paper, the optimization procedure of hungarian method, brute force technique, and greedy approach aims to identify the optimal assignment while considering the uncertainty in the objective values. For job allocations, the results are obtained using these methods and by using arithmetic operations of interval and TrIFNs. Finally, the solution obtained are optimal since the cost values are decreased compared to the existing method.

The succeeding sections gives the construction of the article:
(i) Section 2 describes the fundamental ideas associated with our research are enumerated.
(ii) Section 3 gives an extensive description of the mathematical formulation of IFAP.
(iii) Section 4 discusses the methodology used to solve IFAP using several approaches.
(iv) Section 5 specifies a numerical illustration to examine the concept of this paper.
(v) Section 6 contains the findings and discussions of the current paper.
(vi) Section 7 depicts the conclusion and future work of the present paper.

2. Preliminaries

This section elaborately illustrates the fundamental ideas of FS, IVFS, IFS, IVIFS, ranking interval numbers and new interval arithmetic operations.

2.1. *Fuzzy set [22]*

If $\mathbb{X}$ is the universal set and FS $\tilde{A}$ in $\mathbb{X}$ is interpreted as a set of ordered pairs and $\tilde{A}$ is defined with the equation,

$$\tilde{A} = \{\mathtt{a}, \mu_{\tilde{A}}(\mathtt{a}) : \mathtt{a} \in \mathbb{X},\ \mu_{\tilde{A}}(\mathtt{a}) \in [0,1]\}$$

$\mu_{\tilde{A}}(\mathtt{a})$ represents the grade of membership or Membership Function (MF) of $\mathtt{a}$ in $\tilde{A}$, maps $\mathbb{X}$ to [0,1].

2.2. *Interval-valued Fuzzy set [23]*

IVFS on the Universe $\mathbb{X}$ maps $\tilde{A} : \mathbb{X} \to \mathbb{L}^I$ such that $\mu_{\tilde{A}}(\mathtt{a}) = [\underline{\tilde{A}}(\mathtt{a}), \overline{\tilde{A}}(\mathtt{a})]$ for any $\mathtt{a} \in \mathbb{X} \neq \emptyset$, where $\underline{\tilde{A}} \leq \overline{\tilde{A}}$ and this interval value $\underline{\tilde{A}}, \overline{\tilde{A}} \in \tilde{A} \in \mathbb{X}$. Evidently, a fuzzy set is being assessed if $\underline{\tilde{A}}(\mathtt{a}) = \overline{\tilde{A}}(\mathtt{a})$ for every $\mathtt{a}$ that belongs to $\mathbb{X}$. Thus, FS are specific forms of IVFS. Here, the Lattice $\mathbb{L}^I$ represents $\{[\mathtt{a}_1, \mathtt{a}_2] : \mathtt{a}_1, \mathtt{a}_2 \in [0,1], \mathtt{a}_1 \leq \mathtt{a}_2\}$.

2.3. *Intuitionistic Fuzzy set [24]*

An IFS $\tilde{A}^I$ in $\mathbb{X}$ is described as an object with the form:

$$\tilde{A}^I = \{\mathtt{a}, \mu_{\tilde{A}}(\mathtt{a}), \nu_{\tilde{A}}(\mathtt{a}) : \mathtt{a} \in \mathbb{X}\},$$

where the functions, $\mu_{\tilde{A}}(\mathtt{a}), \nu_{\tilde{A}}(\mathtt{a}) \in [0,1]$ determine the degrees of MF and Non-membership Function (NMF) of the component $\mathtt{a} \in \mathbb{X}$ respectively, and for every $\mathtt{a} \in \mathbb{X}$:

$$0 \leq \mu_{\tilde{A}}(\mathtt{a}) + \nu_{\tilde{A}}(\mathtt{a}) \leq 1$$

Naturally, each FS $\tilde{A}$ may be expressed as $\{\mathtt{a}, \mu_{\tilde{A}}(\mathtt{a}), 1 - \nu_{\tilde{A}}(\mathtt{a}) : \mathtt{a} \in \mathbb{X}\}$. The value $\pi^I_{\tilde{A}}(\mathtt{a}) = 1 - \mu_{\tilde{A}}(\mathtt{a}) - \nu_{\tilde{A}}(\mathtt{a})$ is known as the non-determinacy degree or hesitation margin of the component $\mathtt{a} \in \mathbb{X}$ in the IFS $\tilde{A}^I$.

2.4. *IVIFS [25]*

In the Universe of discourse $\mathbb{X}$, an IVIFS $\tilde{A}^{IV}$ is defined as,

$$\begin{aligned}\tilde{A}^{IV} &= \{\mathtt{a}, \mu_{\tilde{A}}(\mathtt{a}), \nu_{\tilde{A}}(\mathtt{a}) : \mathtt{a} \in \mathbb{X}\} \\ &= \{\mathtt{a}, [\mu^-_{\tilde{A}}(\mathtt{a}), \mu^+_{\tilde{A}}(\mathtt{a})], [\nu^-_{\tilde{A}}(\mathtt{a}), \nu^+_{\tilde{A}}(\mathtt{a})] : \mathtt{a} \in \mathbb{X}\},\end{aligned}$$

where $\mu_{\tilde{A}}(\mathtt{a}) \subseteq [0,1], \nu_{\tilde{A}}(\mathtt{a}) \subseteq [0,1]$ satisfying $0 \leq \mu^+_{\tilde{A}}(\mathtt{a}) \leq \nu^+_{\tilde{A}}(\mathtt{a}) \leq 1$.
Also, we calculate the degree of hesitation for each $\mathtt{a} \in \mathbb{X}$ such as, $\pi^{IV}_{\tilde{A}}(\mathtt{a}) = [1 - \mu^+_{\tilde{A}}(\mathtt{a}) - \nu^+_{\tilde{A}}(\mathtt{a}), 1 - \mu^-_{\tilde{A}}(\mathtt{a}) - \nu^-_{\tilde{A}}(\mathtt{a})]$. **Figure 1** displays the IVIFS's general diagrammatic representation.

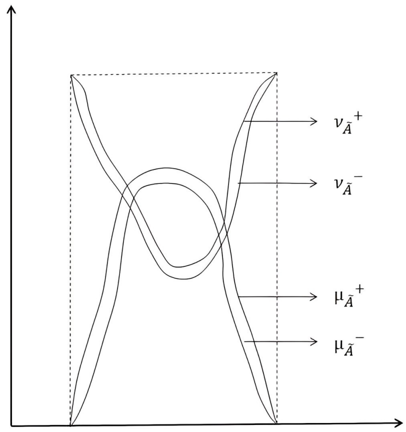


Figure 1. Interval valued IFS

2.5. TrIFN [26]

Let us consider a TrIFNs $\tilde{A}^T = (t_1, t_2, t_3, t_4; \mu_{\tilde{A}})(t_1', t_2, t_3, t_4'; \nu_{\tilde{A}})$ on the real line $\mathbb{R}$, where $0 \leq \mu_{\tilde{A}}, \nu_{\tilde{A}} \leq 1$ and o $\leq \mu_{\tilde{A}} + \nu_{\tilde{A}} \leq 1$ is diagrammatically shown in **Figure 2**, whose MF is,

$$\mu_{\tilde{A}}^T(y) = \begin{cases} \frac{y-t_1}{t_2-t_1}\mu_{\tilde{A}}, & t_1 \leq y < t_2 \\ \mu_{\tilde{A}}, & t_2 \leq y < t_3 \\ \frac{t_4-y}{t_4-t_3}\mu_{\tilde{A}}, & t_3 < y \leq t_4 \\ 0 & Otherwise \end{cases}$$

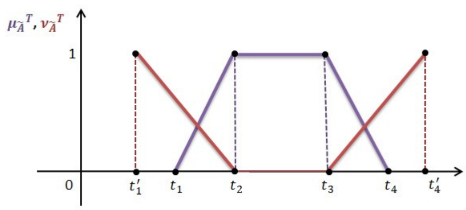


Figure 2. Trapezoidal Intuitionistic fuzzy numbers

The NMF of TrIFN is described as,

$$\nu_{\tilde{A}}^T(y) = \begin{cases} \frac{t_2-y+\nu_{\tilde{A}}(y-t_1')}{t_2-t_1'}\mu_{\tilde{A}}, & t_1' \leq y < t_2 \\ \nu_{\tilde{A}}, & t_2 \leq y < t_3 \\ \frac{y-t_3+\nu_{\tilde{A}}(t_4'-y)}{t_4'-t_3}\mu_{\tilde{A}}, & t_3 < y \leq t_4' \\ 1 & Otherwise \end{cases}$$

2.6. Interval Number [27]

We consider an interval on the real line $\mathbb{R}$ and is displayed by, $I = [i_1, i_2] = \{\mathtt{a} \in \mathbb{R} : i_1 \leq \mathtt{a} \leq i_2 \text{ and } i_1, i_2 \in \mathbb{R}\}$ If $\tilde{I} = i_1 = i_2 = I$, then $\tilde{I} = [i_1, i_2] = I$ is a real number or degenerate interval, then for an interval $\tilde{I} = [i_1, i_2]$, the midpoint and width can be expressed as $\tilde{i}_m = \left(\frac{i_1+i_2}{2}\right), \tilde{i}_w = \left(\frac{i_2-i_1}{2}\right)$ respectively. The midpoint and width of this interval number $\tilde{I}$ can be redefined as follows: $\tilde{I} = [i_1, i_2] = ((\tilde{i}_m), (\tilde{i}_w))$.

2.7. Ranking of Interval numbers [28]

Sengupta et al. presented a easy and effective strategy for connecting any two intervals on real numbers while accounting for decision-makers' preferences. If '$\preceq$' indicates the extended order relation between the pair of intervals $\tilde{p} = [p_1, p_2]$ and $\tilde{q} = [q_1, q_2]$, then

(i) $If(\tilde{p}_m) < (\tilde{q}_m)$, then $\tilde{p} < \tilde{q}$ (or) $\tilde{p}$ is lesser to $\tilde{q}$.
(ii) $If(\tilde{p}_m) > (\tilde{q}_m)$, then $\tilde{p} > \tilde{q}$ (or) $\tilde{p}$ is higher to $\tilde{q}$.
(iii) $If(\tilde{p}_m) = (\tilde{q}_m)$, then $\tilde{p} = \tilde{q}$ (or) $\tilde{p}$ is equal to $\tilde{q}$.
Here $\tilde{p}_m, \tilde{q}_m$ represents the midpoint of the intervals $\tilde{p}, \tilde{q}$ respectively.

Figure 3. Example of an Interval number

The Acceptibility function can be used to formulate overlapping intervals, for example, if we consider two intervals [0.5,0.9] and [0.6,0.8], which is pictured in **Figure 3**, where the midpoints are equal but the intervals are not equal.

On the real line $\mathbb{R}$, I represents the collection of all closed intervals. We define **Acceptibility function** $\mathbb{A}$: I x I $\rightarrow$ [0,∞) such that $\mathbb{A}(\tilde{p} \preceq \tilde{q})$ or $\mathbb{A}_{\preceq} = \frac{\tilde{p}_m - \tilde{q}_m}{\tilde{q}_w + \tilde{p}_w}$, where $\tilde{q}_w + \tilde{p}_w \neq 0$; and $\tilde{p}_m$ and $\tilde{q}_m$ are midpoints of the respected intervals; $\tilde{p}_w$ and $\tilde{q}_w$ are width of the intervals $\tilde{p}$ and $\tilde{q}$. $\mathbb{A}(\tilde{p} \preceq \tilde{q})$ may be analysed as the grade of acceptibility of the first interval $\tilde{p}$ is inferior to the second interval $\tilde{q}$. The grade of acceptability of $\mathbb{A}(\tilde{p} \preceq \tilde{q})$ may be classified and represented as,

$$\mathbb{A}(\tilde{p} \ \preceq \ \tilde{q}) = \begin{cases} = 0 & if \ \tilde{p}_m = \tilde{q}_m, \\ > 0, < 1 & if \ \tilde{p}_m < \tilde{q}_m \\ \geq 1 & if \ \tilde{p}_m < \tilde{q}_m \end{cases}$$

- If $\mathbb{A}(\tilde{p} \ \preceq \ \tilde{q}) = 0$, then the statement '$\tilde{p}$ is inferior to $\tilde{q}$' is not accepted.
- If $0 < \mathbb{A}(\tilde{p} \ \preceq \ \tilde{q}) < 1$, then the assumption is recognised with varying degrees of satisfaction ranging from 0 to 1 (eliminating 0 and 1).
- If $\mathbb{A}(\tilde{p} \ \preceq \ \tilde{q}) \geq 1$, the interpreter $(\tilde{p} \ \preceq \ \tilde{q})$ is accepted.

2.8. New Interval Arithmetic Operations [29]

Ming Ma et al. introduced a new form of fuzzy interval arithmetic that builds on both the location index and the fuzziness index functions. The location index number is considered normal arithmetic, however the fuzziness index function is observed to obey the lattice rule, which is the rule having the lowest upper bound in the lattice L. Let $\tilde{p}, \tilde{q} \in$ L, later we say, $\tilde{p} \vee \tilde{q}$ = max $\{\tilde{p}, \tilde{q}\}$ and $\tilde{p} \wedge \tilde{q}$ = min $\{\tilde{p}, \tilde{q}\}$.

For any two intervals, $\tilde{P} = [p_1, p_2], \tilde{Q} = [q_1, q_2]$, the arithmetic operations on $\tilde{P}$ and $\tilde{Q}$ are well justied as,

$$(i) \tilde{P} + \tilde{Q} = \langle \tilde{p}_m, \tilde{p}_w \rangle + \langle \tilde{q}_m, \tilde{q}_w \rangle = \langle \tilde{p}_m + \tilde{q}_m, max(\tilde{p}_w, \tilde{q}_w) \rangle$$
$$(ii) \tilde{P} - \tilde{Q} = \langle \tilde{p}_m, \tilde{p}_w \rangle - \langle \tilde{q}_m, \tilde{q}_w \rangle = \langle \tilde{p}_m - \tilde{q}_m, max(\tilde{p}_w, \tilde{q}_w) \rangle$$
$$(iii) \tilde{P} \times \tilde{Q} = \langle \tilde{p}_m, \tilde{p}_w \rangle \times \langle \tilde{q}_m, \tilde{q}_w \rangle = \langle \tilde{p}_m \times \tilde{q}_m, max(\tilde{p}_w, \tilde{q}_w) \rangle$$
$$(iv) \tilde{P} \div \tilde{Q} = \langle \tilde{p}_m, \tilde{p}_w \rangle \div \langle \tilde{q}_m, \tilde{q}_w \rangle = \langle \tilde{p}_m \div \tilde{q}_m, max(\tilde{p}_w, \tilde{q}_w) \rangle$$

Here $\tilde{p}_m, \tilde{q}_m$ is the midpoint of the intervals $\tilde{P}, \tilde{Q}$ respectively, whereas $\tilde{p}_w, \tilde{q}_w$ is the width of the intervals $\tilde{P}, \tilde{Q}$.

2.9. *TrIFN Arithmetic Operations [30]*

Let us consider $\tilde{p}$ and $\tilde{q}$ be any two Trapezoidal Fuzzy Numbers (TrFNs), i.e.,

- $\tilde{p} = (p_1, p_2, p_3, p_4)$ = (core of $\tilde{p}$, left spread of $\tilde{p}$, right spread of $\tilde{p}$)
  $\tilde{p} = ([p_2, p_3], \alpha_1, \beta_1)$, core of $\tilde{p} \in \tilde{p}$ and
- $\tilde{q} = (q_1, q_2, q_3, q_4)$ = (core of $\tilde{q}$, left spread of $\tilde{q}$, right spread of $\tilde{q}$)
  $\tilde{q} = ([q_2, q_3], \alpha_2, \beta_2)$, core of $\tilde{q} \in \tilde{q}$.

The arithmetic operations on $\tilde{p}$ and $\tilde{q}$ are well demonstrated and defended as, $* \in \{+, -, \times, \div\}$ as,

$$\begin{aligned}\tilde{p} * \tilde{q} &= \{(\text{core of } \tilde{p} * \text{core of } \tilde{q}, \max(\alpha_1, \alpha_2), \max(\beta_1, \beta_2))\}\\ &= \{([p_2, p_3] * [q_2, q_3], \max(\alpha_1, \alpha_2), \max(\beta_1, \beta_2))\}\end{aligned}$$

3. Trapezoidal Intuitionistic fuzzy AP using Interval Arithmetic

An Assignment model is an unique event of transportation model in which each supply is assigned to a demand and each demand should get connected. Allocating jobs can be attained by using various methodologies in which the costs $\tilde{c}_{ij}$ are represented in TrFNs with MF and NMF. The main purpose of the problem is used to identify the optimal assignment allotment for the parameters including cost, profit and time which is identified priorly. In many realistic conditions, the factors like cost, time may not be known precisely earlier since there may be many unmanageable parameters like human prediction and market variation. Hence, to overcome these circumstances it is easy to consider in interval rather than crisp values.

Table 1. General representation of Assignment Problem

	Worker 1	**Worker 2**	**Worker 3**	.....	**Worker n**	$\tilde{a}_i$
Job 1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\tilde{c}_{13}$	.....	$\tilde{c}_{1n}$	1
Job 2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	$\tilde{c}_{23}$	.....	$\tilde{c}_{2n}$	1
Job 3	$\tilde{c}_{31}$	$\tilde{c}_{32}$	$\tilde{c}_{33}$	.....	$\tilde{c}_{3n}$	1
....	....	....	....	....	....	....
Job n	$\tilde{c}_{n1}$	$\tilde{c}_{n2}$	$\tilde{c}_{n3}$	.....	$\tilde{c}_{nn}$	1
$\tilde{b}_j$	1	1	1	.....	1	

Let us consider the circumstance in which 'r' jobs are assigned to 's' workers and $\tilde{c}_{ij}$ is the cost of alloting i^{th} job (i=1,2,3,...r) to j^{th} worker (j=1,2,3,....s). The main objective of assigning jobs to workers is to reduce the total cost or to increase the total profit. The source and destination is considered as jobs and workers respectively, the supply at every source is 1, i.e., $\tilde{a}_i = 1 \; \forall \; i$ and the demand at every destination is 1, i.e., $\tilde{b}_j = 1 \; \forall \; j$ i.e., single job is assigned to single worker. If the goal of an AP is to reduce a particular cost, time, or quantity, for example, this type of AP is known as a single objective AP. Alternatively, if the goal is to reduce all factors such as cost, time, quantity, and so on, this is referred to as multi objective AP. An AP is characterized by means of n $\times$ n matrix and is shown in **Table 1**.

3.1. *Mathematical formulation*

Let the i^{th} work is allotted to the j^{th} worker and is represented by $\tilde{x}^{ij}$ and $\tilde{c}^{ij}$ be the corresponding cost of i^{th} job to the j^{th} person. Since our goal is to reduce the complete cost for executing all jobs, mathematically an IFAP can be expressed as,

$$\tilde{x}_{ij} = \begin{cases} 1, & \text{if } i^{th} \; job \; is \; assigned \; to \; j^{th} \; worker \\ 0, & \text{otherwise} \end{cases}$$

Mathematically, an assignment model can be characterized as,

$$\text{Maximize or Minimize } \tilde{z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}^{ij} \tilde{x}^{ij}$$

subject to,

$$\sum_{i=1}^{n} \tilde{x}^{ij} = 1, \text{ j = 1, 2, 3,...., n}$$

$$\sum_{j=1}^{n} \tilde{x}^{ij} = 1, \text{ i = 1, 2, 3,...., n}$$

$$\text{and } \tilde{x}^{ij} = 0 \text{ } or \text{ } 1 \ \forall \text{ i, j}$$

where $\tilde{c}^{ij}$ is the total time taken in assigning i^{th} job to j^{th} worker.
and $\tilde{x}_{ij}$ is the decision variable which means that the allotment is done from i^{th} job to j^{th} worker.
Some features of the Assignment model are,
(i) The entire cost matrix is a square matrix in which the number of rows is equal to number of column i.e., m = n. If m $\neq$ n, it is essential to construct m = n by including jobs or workers.
(ii) The entire number of possible solutions for n $\times$ n AP is always n!.

4. Computational method

In this section, we can compute the optimal solution of IFAP by using various methods such as HM, BFM and GM.

4.1. Hungarian methodology

The Costs $[\tilde{c}^{ij}]$ are displayed as TrFNs whereas the MF and NMF are described in IVIFNs. The following steps perfom the methodology for solving IFAP:
Step 1: Perform the specified problem in the Intuitionistic Fuzzy Cost Table (IFCT). If the amount of origins and destinations are equal, proceed to step 2; otherwise, add a duplicate origin or destination such that the given IFCT becomes a square matrix.
Step 2: Find the minimum and next minimum values in the row and subtract those elements of the same row-line.
Step 3: Following the resulting IFAP in step 2, the same procedure is followed for column. After applying this, every row and column has at the minimum of one zero.
Step 4: From the altered IFCT from step 3, explore the IFAP optimal solution as displays below,
(a) Scan the rows clearly up to a row with only one 0 is located. Allot that 0 to the corresponding row and strike off all extra zeros in the corresponding column. Pursue this step for all the rows.
(b) Reiterate the same way for every column of minimized IFCT.
(c) In case, a row or column with two or many zeros are found, allot a random 0 for any of the zeros and strike off all other zeros of that row or column. Reproduce (a) via (c) successively till the striking stops.
Step 5: If the result obtained as the number of allotments is identical to n, then the optimal solution is satisfied. If the total amount of allotments is less than n, proceed to step 6.
Step 6: Create a few parallel or perpendicular lines as possible to encircle all of the obtained IFAP zeros. The steps below can be used to finish this,
(i) Find the rows where there are no allocated zeros.
(ii) Identify the columns in the spotted rows that contain zeros.
(iii) Find the rows in the spotted columns that don't have any assigned zeros.
(iv) Till the chain of spots is broken, repeat steps (ii) and (iii). Draw lines through all of the columns and rows that are not yet covered. The required minimum number of lines is enabled in this way.
Step 7: Raise the newly modified IFCT in the following manner:
Find the reduced matrix's smallest element that is not covered by any lines. Add that element to all the labelled elements, then subtract it from all the unlabeled elements to get the possible elements at the intersection of any two lines.
Step 8: Repeat steps 6 to 8 until the specified IFAP's optimal outcome is achieved.

4.2. Brute force method

- The cost of all assignment values can be calculated by this Brute force method, which is quite complicated method mainly for huge assignments.
- Unlike some of the other methods, this method is appropriate to a broad range of problems.
- A brute-force method provides an important theoretical or educational basis.
- If we have mxn assignment matrix with m is equal to n, there are n possibility for the initial assignment and n - 1 possibility for the next assignment, respectively and finally it gives n!. We have to find the matrix of the given assignment problem.
- If m $\times$ n (m = n) is 3, then the assignments is 3! which is 6. The iterative numerals for the Brute force method is shown in **Table 2**:

Table 2. Iterations of Brute force method

Size of the table	Result	Iteration numbers
2×2	2!	2
3×3	3!	6
4×4	4!	24
5×5	5!	120
6×6	6!	720
7×7	7!	5040
8×8	8!	40320
9×9	9!	362880
10×10	10!	3628800
. .	. .	. .
100×100	100!	9.33×10^{157}

After implementing the iterations, we have to find the minimum of the solutions and hence the brute force method is exhibited.

4.3. Greedy method

Greedy method fundamentally approaches the best decision by selecting the best choice available at that occasion in identifying a best solution. Accordingly, the steps for calculating GM is to select the lowest cost assigned to the employee as the initial assignment, then pick the next least cost as the second assignment, and so forth upto all the jobs have been allotted. The Greedy algorithm can be solved by involving the following steps:
Step 1: Accomplish the given problem in the IFCT. Detect the least element from the problem and delete the corresponding row and column where it has been taken from.
Step 2: Analyze the second least element and remove the corresponding row and column from where it has been located.
Step 3: Reproduce the process as far as all the rows and columns have been removed.
Step 4: The selected least values produces the optimal solutions for the given problem.

5. Numerical Illustration

To exhibit the projected algorithm, we are considering an IFAP which is extracted from [21], alongside the rows are represented with 4 employees via A,B,C,D and the columns as 4 jobs via job_1, job_2, job_3, job_4 which is given in **Table 3**. Thus, the goal is to assign various jobs to individuals who are experts in specialised fields. The cost model $[\tilde{c}^{ij}]$ is described as TrIFNs. The problem's main goal is to determine the best solution in order to reduce the overall cost of assigning jobs to the deserving persons.

Table 3. Trapezoidal intuitionistic fuzzy cost table

Employees	**Job 1**	**Job 2**	**Job 3**	**Job 4**
A	(5,8,10,13;0.5,0.1)	(8,10,12,14;0.7,0.2)	(8,11,13,16;0.5,0.3)	(8,10,12,14;0.7,0.2)
B	(3,5,7,9;0.7,0.1)	(1,2,4,5;0.4,0.3)	(4,7,9,13;0.7,0.1)	(2,4,6,8;0.8,0.1)
C	(5,6,8,9;0.6,0.1)	(2,5,7,10;0.7,0.1)	(10,12,14,16;0.6,0.2)	(8,10,12,14;0.7,0.2)
D	(5,6,8,9;0.6,0.1)	(7,9,11,13;0.7,0.1)	(7,9,11,13;0.7,0.1)	(5,8,10,13;0.5,0.1)

The grades of the crisp MF and NMF are converted into interval numbers, since it can depict the possible value range of information as it is not done in crisp. This process is done by subtracting and adding the current number in which each and every number lies within the interval value.

The TrFNs are represented as $\tilde{p} = (p_1, p_2, p_3, p_4)$ = (core of $\tilde{p}$, left spread of $\tilde{p}$, right spread of $\tilde{p}$)= $([p_2, p_3], \alpha_1, \beta_1)$. Currently, the given IVIFAP is written as follows and is shown in **Table 4**.

Table 4. Trapezoidal Intuitionistic fuzzy numbers in terms of core

Employees	**Job 1**	**Job 2**	**Job 3**	**Job 4**
A	([8,10],3,3); [0.4,0.6][0,0.2]	([10,12],2,2); [0.6,0.8][0.1,0.3]	([11,13],3,3); [0.4,0.6][0.2,0.4]	([10,12],2,2); [0.6,0.8][0.1,0.3]
B	([5,7],2,2); [0.6,0.8][0,0.2]	([2,4],1,1); [0.3,0.5][0.2,0.4]	([7,9],3,4); [0.6,0.8][0,0.2]	([4,6],2,2); [0.7,0.9][0,0.2]
C	([6,8],1,1); [0.5,0.7][0,0.2]	([5,7],3,3); [0.6,0.8][0,0.2]	([12,14],2,2); [0.5,0.7][0.1,0.3]	([10,12],2,2); [0.6,0.8][0.1,0.3]
D	([6,8],1,1); [0.5,0.7][0,0.2]	([9,11],2,2); [0.6,0.8][0,0.2]	([9,11],2,2); [0.6,0.8][0,0.2]	([8,10],3,3); [0.4,0.6][0,0.2]

Let's set all the parameters for the interval as $\tilde{P} = [p_1, p_2]$ and $\tilde{Q} = [q_1, q_2]$ in regards to midpoint and width as $\tilde{P} = \langle(\tilde{p}_m), (\tilde{p}_w)\rangle$ and $\tilde{Q} = \langle(\tilde{q}_m), (\tilde{q}_w)\rangle$ which is mentioned in **Definition 6** and is indicated in **Table 5**.

Table 5. TrIFNs in terms of midpoint and width

Employees	**Job 1**	**Job 2**	**Job 3**	**Job 4**
A	($\langle 9, 1\rangle$,3,3); [0.4,0.6][0,0.2]	($\langle 11, 1\rangle$,2,2); [0.6,0.8][0.1,0.3]	($\langle 12, 1\rangle$,3,3); [0.4,0.6][0.2,0.4]	($\langle 11, 1\rangle$,2,2); [0.6,0.8][0.1,0.3]
B	($\langle 6, 1\rangle$,2,2); [0.6,0.8][0,0.2]	($\langle 3, 1\rangle$,1,1); [0.3,0.5][0.2,0.4]	($\langle 8, 1\rangle$,3,4); [0.6,0.8][0,0.2]	($\langle 5, 1\rangle$,2,2); [0.7,0.9][0,0.2]
C	($\langle 7, 1\rangle$,1,1); [0.5,0.7][0,0.2]	($\langle 6, 1\rangle$,3,3); [0.6,0.8][0,0.2]	($\langle 13, 1\rangle$,2,2); [0.5,0.7][0.1,0.3]	($\langle 11, 1\rangle$,2,2); [0.6,0.8][0.1,0.3]
D	($\langle 7, 1\rangle$,1,1); [0.5,0.7][0,0.2]	($\langle 10, 1\rangle$,2,2); [0.6,0.8][0,0.2]	($\langle 10, 1\rangle$,2,2); [0.6,0.8][0,0.2]	($\langle 9, 1\rangle$,3,3); [0.4,0.6][0,0.2]

5.1. Hungarian method

This algorithm signifies the costs of allocating every job to every employee. The algorithm then goes through a number of iterations to determine the best job assignment. After carrying out the procedures from tables 4 and

5, the row reduction and the column reduction is performed by subtracting the minimum element from the other elements; the resulting cost table is **Table 6**.

Table 6. Allocation of Cost of jobs to the employees after reductions

Employees	**Job 1**	**Job 2**	**Job 3**	**Job 4**
A	0	$(\langle 2,1\rangle$,3,3); [0.6,0.8][0.1,0.3]	0	0
B	$(\langle 3,1\rangle$,2,2); [0.6,0.8][0,0.2]	0	$(\langle 5,1\rangle$,3,4); [0.6,0.8][0,0.2]	0
C	$(\langle 1,1\rangle$,3,3); [0.5,0.7][0,0.2]	0	$(\langle 7,1\rangle$,3,3); [0.5,0.7][0.1,0.3]	$(\langle 5,1\rangle$,3,3); [0.6,0.8][0.1,0.3]
D	0	$(\langle 3,1\rangle$,2,2); [0.6,0.8][0,0.2]	0	0

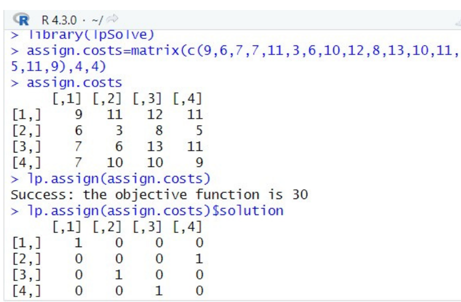
To wrap the zeros in the cells, find the essential number of parallel and perpendicular lines. Here the order of cost matrix in our required Hungarian Assignment problem is four, and so it requires four lines to cover the area of zeros. After covering the zeros, the optimal allocation for AP is shown in **Table 7** and allocation is, A $\rightarrow$ Job 1, B $\rightarrow$ Job 4, C $\rightarrow$ Job 2, D $\rightarrow$ Job 3.

Table 7. Final Allocation of cost of jobs to the employees using HM

Employees	**Job 1**	**Job 2**	**Job 3**	**Job 4**
A	$\boxed{0}$	$(\langle 2,1\rangle$,3,3); [0.6,0.8][0.1,0.3]	0	0
B	$(\langle 3,1\rangle$,2,2); [0.6,0.8][0,0.2]	0	$(\langle 2,1\rangle$,3,4); [0.6,0.8][0,0.2]	$\boxed{0}$
C	$(\langle 1,1\rangle$,3,3); [0.5,0.7][0,0.2]	$\boxed{0}$	$(\langle 4,1\rangle$,3,3); [0.5,0.7][0.1,0.3]	$(\langle 3,1\rangle$,3,3); [0.6,0.8][0.1,0.3]
D	0	$(\langle 3,1\rangle$,2,2); [0.6,0.8][0,0.2]	$\boxed{0}$	0

From the above table, the optimal allocation of Trapezoidal Intuitionistic Fuzzy Assignment Problem (TrIFAP) is obtained by substituting the alloted cells into the initial problem and the optimum solution is resulted as,

$= (\langle 9,1\rangle, 3, 3)[0.4, 0.6][0, 0.2] + (\langle 5,1\rangle, 2, 2)[0.7, 0.9][0, 0.2] + (\langle 6,1\rangle, 3, 3)[0.6, 0.8][0, 0.2] +$
$(\langle 10,1\rangle, 2, 2)[0.6, 0.8][0, 0.2]$
$= (\langle 30,1\rangle, 3, 3)[0.4, 0.6][0, 0.2]$
$= ([29, 31], 3, 3)[0.4, 0.6][0, 0.2]$
$= (26, 29, 31, 34)[0.4, 0.6][0, 0.2]$



```
R 4.3.0 · ~/
> library(lpSolve)
> assign.costs=matrix(c(9,6,7,7,11,3,6,10,12,8,13,10,11,
5,11,9),4,4)
> assign.costs
     [,1] [,2] [,3] [,4]
[1,]    9   11   12   11
[2,]    6    3    8    5
[3,]    7    6   13   11
[4,]    7   10   10    9
> lp.assign(assign.costs)
Success: the objective function is 30
> lp.assign(assign.costs)$solution
     [,1] [,2] [,3] [,4]
[1,]    1    0    0    0
[2,]    0    0    0    1
[3,]    0    1    0    0
[4,]    0    0    1    0
```

Figure 4. Solution of AP using HM and R Program

As a result, the degrees of MF and NMF are displayed in IVIFNs and the cost is represented in TrFNs and the optimal solution for the IFAP is produced as (26,29,31,34)[0.4,0.6][0,0.2]. It is obtained using R programming and the code is given in **Figure 4**. Here, the midpoint is considered as the cost value and width is as the maximum value.

5.2. Brute force method

This process is a clear-cut method used as benchmark for evaluating the efficacy of optimization algorithms. From the table 6, the original formulation of LPP representing the midpoint is,

$$\text{Min } \tilde{z} = 9x^{11} + 11x^{12} + 12x^{13} + ... + 9x^{44}$$

subject to the constraints,

$$\begin{aligned} x^{11} + x^{12} + x^{13} + x^{14} = 1 \\ x^{21} + x^{22} + x^{23} + x^{24} = 1 \\ x^{31} + x^{32} + x^{33} + x^{34} = 1 \\ x^{41} + x^{42} + x^{43} + x^{44} = 1 \\ x^{11} + x^{21} + x^{31} + x^{41} = 1 \\ x^{12} + x^{22} + x^{32} + x^{42} = 1 \\ x^{13} + x^{23} + x^{33} + x^{43} = 1 \\ x^{14} + x^{24} + x^{34} + x^{44} = 1 \end{aligned}$$

where $x^{ij} = 1$, i=1,2,3,4 and j=1,2,3,4
i.e., the supply point i is converged to meet the demand point j.

The optimal solution for the given problem is obtained for 4 $\times$ 4 matrix is shown in **Table 8**, but for m $\times$ n matrix, it is easy to solve by using R programming software.

Table 8. Results of BFM

Columns				**Solution**
P1	P2	P3	P4	9+3+13+9=34
P1	P2	P4	P3	9+3+11+10=33
P1	P4	P3	P2	9+5+13+10=37
⋮	⋮	⋮	⋮	⋮
P2	P4	P3	P1	11+5+13+7=36
P2	P3	P4	P1	11+8+11+7=37
P2	P1	P3	P4	11+6+13+9=39
⋮	⋮	⋮	⋮	⋮
P3	P1	P2	P4	12+6+6+9=33
P3	P2	P1	P4	12+3+7+9=31
P3	P4	P2	P1	12+5+6+7=30 **(min)**
⋮	⋮	⋮	⋮	⋮
P4	P1	P2	P3	11+6+6+10=33
P4	P1	P3	P2	11+6+13+10=40
P4	P2	P3	P1	11+3+13+7=34

The optimal allocation of TrIFAP is obtained by replacing the alloted cells into the primary problem and the optimum solution is resulted as,

$= (\langle 12, 1\rangle, 3, 3)[0.4, 0.6][0.2, 0.4] + (\langle 5, 1\rangle, 2, 2)[0.7, 0.9][0, 0.2] + (\langle 6, 1\rangle, 3, 3)[0.6, 0.8][0, 0.2] +$
$(\langle 7, 1\rangle, 1, 1)[0.5, 0.7][0, 0.2]$

$= (\langle 30, 1\rangle, 3, 3)[0.4, 0.6][0.2, 0.4]$
$= ([29, 31], 3, 3)[0.4, 0.6][0.2, 0.4]$
$= (26, 29, 31, 34)[0.4, 0.6][0.2, 0.4]$
Thus, the various degrees of MF and NMF is shown in IVIFNs and the cost is represented in TrFNs and the optimal solution for the AP is produced as (26,29,31,34)[0.4,0.6][0.2,0.4].

5.3. Greedy method

After getting the values in midpoint and width (from Table 5), the greedy method is proceeded and the values obtained as is exhibited in **Table 9**.

Table 9. Allocation of jobs to employees using GM

Employees	Job 1	Job 2	Job 3	Job 4
A	$(\langle 9, 1\rangle$,3,3); [0.4,0.6][0.0,0.2]	$(\langle 11, 1\rangle$,2,2); [0.6,0.8][0.1,0.3]	$\boxed{(\langle 12, 1\rangle,3,3);}$ [0.4,0.6][0.2,0.4]	$(\langle 11, 1\rangle$,2,2); [0.6,0.8][0.1,0.3]
B	$(\langle 6, 1\rangle$,2,2); [0.6,0.8][0,0.2]	$\boxed{(\langle 3, 1\rangle,1,1);}$ [0.3,0.5][0.2,0.4]	$(\langle 8, 1\rangle$,3,4); [0.6,0.8][0,0.2]	$(\langle 5, 1\rangle$,2,2); [0.7,0.9][0,0.2]
C	$\boxed{(\langle 7, 1\rangle,1,1);}$ [0.5,0.7][0,0.2]	$(\langle 6, 1\rangle$,3,3); [0.6,0.8][0,0.2]	$(\langle 13, 1\rangle$,2,2); [0.5,0.7][0.1,0.3]	$(\langle 11, 1\rangle$,2,2); [0.6,0.8][0.1,0.3]
D	$(\langle 7, 1\rangle$,1,1); [0.5,0.7][0,0.2]	$(\langle 10, 1\rangle$,2,2); [0.6,0.8][0,0.2]	$(\langle 10, 1\rangle$,2,2); [0.6,0.8][0,0.2]	$\boxed{(\langle 9, 1\rangle,3,3);}$ [0.4,0.6][0,0.2]

The optimal allocation of AP using GM is finalised as,
$= (\langle 12, 1\rangle, 3, 3)[0.4, 0.6][0.2, 0.4] + (\langle 3, 1\rangle, 1, 1)[0.3, 0.5][0.2, 0.4] + (\langle 7, 1\rangle, 1, 1)[0.5, 0.7][0, 0.2] +$
$(\langle 9, 1\rangle, 3, 3)[0.4, 0.6][0, 0.2]$
$= (\langle 31, 1\rangle, 3, 3)[0.3, 0.5][0.2, 0.4]$
$= ([30, 32], 3, 3)[0.3, 0.5][0.2, 0.4]$
$= (27, 30, 32, 35)[0.3, 0.5][0.2, 0.4]$
Hence, the degrees of MF and NMF are displayed in IVIFNs and the cost is represented in trapezoidal numbers and the optimal solution for the Assignment problem is (27,30,32,35)[0.3,0.5] [0.2,0.4].

6. Results and Discussions

The original problem [21] is compared with the various methods in the proposed paper. The optimum result which we obtained in the existing method and by using HM, BFM and GM is given in **Table 10**.

Table 10. Comparison of the results

S. No.	Methods	Optimal results
1.	Existing method	(21, 28, 36, 43; 0.4, 0.3)
2.	Proposed method using HM	(26, 29, 31, 34; [0.4,0.6] [0, 0.2])
3.	Proposed method using BFM	(26, 29, 31, 34; [0.4,0.6] [0.2, 0.4])
4.	Proposed method using GM	(27, 30, 32, 35; [0.3,0.5] [0.2, 0.4])

- The optimal assignment of the existing problem is, A $\rightarrow$ Job 1, B $\rightarrow$ Job 2, C $\rightarrow$ Job 3, D $\rightarrow$ Job 4 and the corresponding optimal Trapezoidal Intuitionistic Fuzzy Cost is (21,28,36,43; 0.4,0.3).

- The optimal assignment of allocating jobs to workers which is acquired using HM is, A $\rightarrow$ Job 1, B $\rightarrow$ Job 2, C $\rightarrow$ Job 3, D $\rightarrow$ Job 4 and the corresponding Interval-valued Intuitionistic Fuzzy Cost (IVIFC) is (26,29,31,34)[0.4,0.6][0,0.2].

The suggested optimal solutions of the HM and existing approach are shown in **Figure 5**. The values of existing and HM are compared and it is displayed using MATLAB.

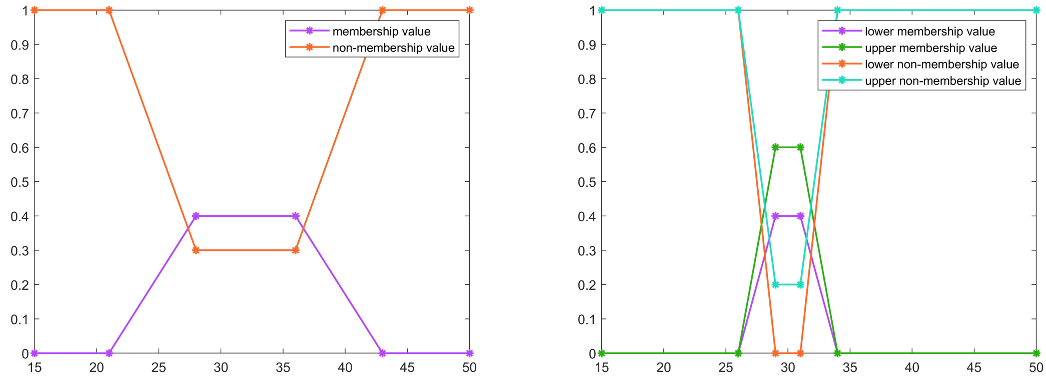


Figure 5. Comparison of results between existing and proposed HM

The cost using HM is from 26 to 34 and is expressed in terms of intervals with the acceptance rate (MF) of 0.4 to 0.6 and rejection rate (NMF) of 0 to 0.2. In this case, the IVIFC is minimised compared to the optimal result produced using the existing approach, where the trapezoidal cost is from 21 to 43. With the help of BFM and GM, the optimum result of IVIFC is shown in **Figure 6**.

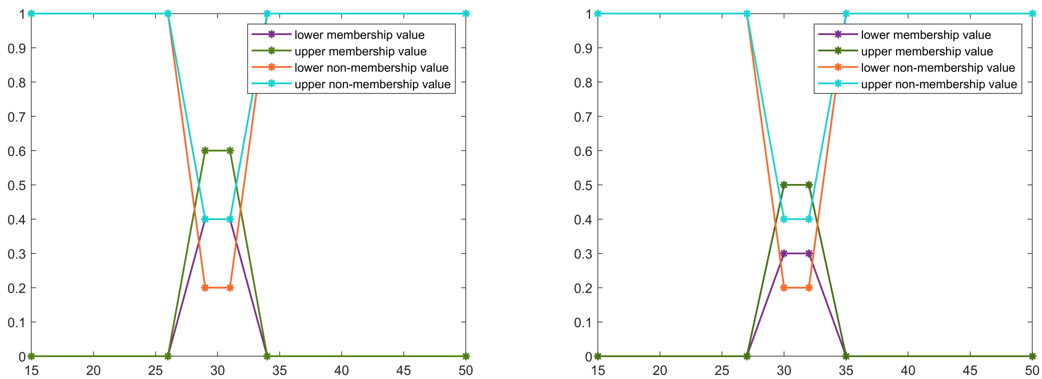


Figure 6. Resuts using BFM and GM

By using BFM, the optimal result acquired is (26, 29, 31, 34; [0.4, 0.6][0.2, 0.4]). Here, the cost is in TrFNs ranging from 26 to 34 with the acceptance rate of 40% to 60% and rejection is 20% to 40%.
By using GM, the optimal result acquired is (27, 30, 32, 35; [0.3, 0.5][0.2, 0.4]). The price ranges from 27 to 35, with an acceptance rate of 30% to 50% and a rejection rate of 20% to 40%.

- HM provides the most effective optimal solution of the three approaches since its rejection rate is lower than the other methods.

- The decision-makers can choose quickly which job should be assigned to which employee by using HM and it is found in the original problem, i.e., Job 1 $\rightarrow$ A, Job 2 $\rightarrow$ B, Job 3 $\rightarrow$ C and Job 4 $\rightarrow$ D.

6.1. Advantages and Limitations

- By implementing the suggested approaches, we have reduced the loss as the cost of assigning jobs is lower when compared with the existing method.
- The techniques presented in this paper can solve IVIFAP quickly and are easy to use.
- Therefore, assigning tasks or projects to people in human resource management might contain uncertainty linked to employee talents, preferences, and task needs. IVIFAP can assist in the creation of more accurate and adaptable job assignments.
- When dealing with large assignment matrices, solving IVIFAP might be difficult. IVIFN optimisation algorithms can be more complex than those used in ordinary crisp or fuzzy assignment situations.
- Accurate IVIFNs from real-world data can be difficult to obtain. Acquiring precise interval estimates for membership and non-membership degrees can be challenging in many real-life situations, resulting in unsatisfactory solutions.

7. Conclusion

IVIFAP recognises the complexity and provides a more realistic representation. IFNs are used since it represents the uncertainty and imprecision more expressively than crisp or fuzzy numbers. The IVIFAP with cost value as TrFNs is formulated by using new arithmetic operations of interval values. The major intention is to decrease the cost by allocating the jobs to workers using various methods such as HM, BFM and GM. These methods are solved using R programming to find optimal solutions. As a conclusion, the comparison is made and the results obtained in this proposed paper are optimum than the existing method and are shown in Figures 5 and 6 with the help of MATLAB. Finally, it is evident that HM is effective among other methods since the acceptance rate is 40% to 60% and rejection is 0% to 20%. In the future, there is the possibility of expanding into a Neutrosophic Fuzzy environment.

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